Number Systems & Codes

1. Perform the following operations in 2's complement method (use 8 bit representation)
   (a) $(52)_{10} - (23)_{10}$
   (b) $(23)_{10} - (52)_{10}$
   (c) $(52)_{10} - (-23)_{10}$
   (d) $(-52)_{10} - (-23)_{10}$
   (e) $(-77)_{10} + (-122)_{10}$

Ans. (a) $\quad$ **$(52)_{10} - (23)_{10}$ can be written as**

52	**Minuend**
-23	**Subtrahend**
+29	

52 =	**0011 0100**	**Binary**
-23 =	**+ 1110 1001**	**2's complement form**
	1 0001 1101	

carry

Discard carry, the answer is positive and in binary form (+29)

(b) $\quad$ **$(23)_{10} - (52)_{10}$**

23
-52
-29

+23 =	**0001 0111**	**Binary**
-52 =	**+ 1100 1100**	**2's complement form**
	1110 0011	

No carry is generated so the answer is negative and in 2's complement form so take another 2's complement to get the result

11100011 $\xrightarrow{\text{2's complement}}$ **$(00011101)_2 = (-29)_{10}$**

(c) $\quad$ **$(52)_{10} - (-23)_{10} \quad = \quad (52)_{10} + (+23)$**

Both numbers are positive so add the numbers in binary

52	**0011 0100**	**Binary**
+23	**0001 0111**	**Binary**
75	**0100 1011**	**$(75)_{10}$**

(d) -52 – 23 = -52 + (-23)

-52	**1100 1100**
+(-23)	**1110 1001**
-75	**1 1011 0101**

Carry

Ignore carry the result is in 2's complement form and –ve

Takes 2's complement of 10110101 $\xrightarrow{\text{2's complement}}$ $(01001011)_2 = (-75)_{10}$

Note if two operands are of the same sign as in this case the sign bit of operand is to be compared with the sign bit for result (MSB). Here it is 1 in all as indicated the result is ok

If sign bits of operand and results are not same then there is a problem of over flow i.e. result cannot be accommodated using eight bits and is to be interpreted suitably.

The result in this case will consist of 9 bits i.e. carry and eight bits, carry bit will give sign of the number.

(e) add -77 and -122

-77	**1011 0011**	**2's complement**
+(-122)	**1000 0110**	**2's complement**
	1 0011 1011	

Overflow

Note that MSB of operands is 1 while that of result is 0 so there is overflow. It occurs when operation results are beyond the capacity of storage registers. So look for the change in sign bit. The result is 2's complement of 00111001

GATESTUDY.COM

i.e. 11000111 (-199)

Range of numbers:

Note that positive numbers with 2's complement ------- $(2^{n-1} -1)$

Negative numbers -------- -2^{n-1}

i.e. +127 and -128 (for 8 bit)

Here, the number is beyond this range (-199) so it has to be interpreted suitably.

2. Consider the operation 24 + 17 = 40. Find the correct base of number so that the operation is correct?

Ans. **Let the base be b**

$$\mathbf{2 \times b^1 + 4 \times b^0 + 1 \times b^1 + 7 \times b^0 = 4 \times b^1 + 0 \times b^0}$$

Or, $\mathbf{2b + 4 + b + 7 = 4b}$

Or, **b = 11**

Base is 11

3. Find F's complement of $(2BFD)_{16}$

Ans. **F's complement of $(2BFD)_{16}$**

Is

(i)

$$\begin{array}{r} \text{F F F F} \\ \underline{-\text{2 B F D}} \\ \text{D 4 0 2} \end{array}$$

(ii)Add 1 to the above answer

$$\begin{array}{r} \text{D 4 0 2} \\ \underline{+1} \\ \text{D 4 0 3} \end{array}$$

F's complement of

$(2BFD)_{16} = (D403)_{16}$

GATESTUDY.COM

4. The subtraction of a binary number Y from another binary number X, done by adding the 2's complement of Y to X, results in a binary number without overflow. This implies that the result is:
   (a) negative and is in normal form
   (b) negative and is in 2's complement form
   (c) positive and is in normal form
   (d) positive and is in 2's complement form

[Gate 1987: 1 Mark]

Ans. (b)

As per the rules of 2's complement addition the resulting number will be negative and is in 2's complement form

5. An equivalent 2's complement representation of the 2's complement number 1101 is
   (a) 110100
   (b) 001101
   (c) 110111
   (d) 111101

[Gate 1988: 1 Mark]

Ans. (d)

The 2's complement number is given 1101which is in 4 bit representation. While the given options are is 6 bits taking 2's complement of the number we will get the corresponding positive number (magnitude). So 2's complement of 1101 is 0011 i.e. $(+3)_{10}$ now represent this in six bit by adding zeros.

i.e. 0011 → 000011 (Bits are extended as per MSB if 0, zeros added if 1, 1 is added)

Taking 2's complement to get the number 111101

6. The 2's complement representation of – 17 is
   (a) 101110
   (b) 101111
   (c) 111110
   (d) 110001

[Gate 2001: 1 Mark]

Ans. (b)

We have to determine 2's complement of - 17

So we determine the number the +17in binary i.e.

$(+17)_{10}$ = 10001

Note that the above number is in 5 bits while the operations given are in 6 bits i.e.

10001 → 010001 (number positive so sign bit is 0)

Now taking 2's complement, we get 101111

7. 4-bit 2's complement representation of a decimal number is 1000. The number is
   (a) +8
   (b) 0
   (c) -7
   (d) -8

[Gate 2002: 1 Mark]

Ans. (d)

Given: 2's complement of a decimal number is represented in 4 bit is 1000

Take another 2's complement to get +ve number (magnitude) the number is

1000 → $(8)_{10}$

So the given number is (-8)

8. The range of signed decimal number that can be represented by 6-bit 1's complement number is
   (a) -31 to +31
   (b) -63 to +63
   (c) -64 to +63
   (d) -32 to 31

[Gate 2004: 1 Mark]

Ans. (a)

The 1's complement number is a 6 bit number. We have to find the maximum and minimum number which can be represented by it.

When number is negative then the maximum value can be 111111 $(-31)_{10}$

Here MSB is for sign.

Maximum positive number is 0 11111 $(+31)_{10}$

GATESTUDY.COM

9. 2’s complement representation of a 16-bit number (one sign bit and 15 magnitude bits) if FFFF. Its magnitude in decimal representation is

(a) 0
(b) 1
(c) 32, 767
(d) 65, 535

[Gate 1993: 1 Mark]

Ans. (b)

Given: 2’s complement representation of a 16 bit number

F F F F

i.e. 1111 1111 1111 1111

If we take another complement we will get the positive number or magnitude.

2’s complement 0000 0000 0000 0001

So it is 1 decimal

10. A signed integer has been stored in byte using the 2’s complement format. We wish to store the same integer in a 16 bit word. We should

(a) Copy the original byte to the less significant byte of the word and fill the more significant byte with zeros.
(b) Copy the original byte to the more significant byte of the word and fill the less significant byte with zeros.
(c) Copy the original byte to the less significant byte of the word and make each bit of the more significant byte equal to the most significant bit of the original byte.
(d) Copy the original byte to the less significant bytes well as the more significant byte of the word.

[Gate 1997: 1 Mark]

Ans. Given, A signed integer has been stored is a single byte using 2’s complement format.

This has to be stored in a 16 bit word. The procedure will be to copy original byte to less significant byte of the word and make each bit of most significant byte equal to the most significant bit of the original byte.

If number is negative the MSB will be 1 so make all bits as 1 in most significant byte.

If MSB is 0 then number is positive, then make all bits as zero.

GATESTUDY.COM