Boolean algebra And Logic Simplifications

In1854 George Boole Introduced systematic treatment of logic and developed an algebra called Boolean algebra.

In 1938 Shannon introduced two valued Boolean algebra called switching algebra.

Some postulates were formulated by Huntington in 1904.

Two valued Boolean algebra is defined on set of two elements

$B = \{0, 1\}$ $w$ith rules for binary operators (+) and (.). It is also called switching algebra.

Duality

An important property of Boolean algebra is called duality principle.

To find the dual of a particular expression we interchange OR and AND operators and replace 1's by 0's and 0's by 1's

Duality Property

Operator	Dual
AND	OR
OR	AND
0	1
1	1

Some of the important laws of Boolean algebra are given in the following table:

S.No.	Name	Boolean laws	Dual
1.		$A + 0 = A$	$A.1 = A$
2.		$A + \bar{A} = 1$	$A.\bar{A} = 0$
3.	Indempotence (sameness)	$A + A = A$	$A.A = A$
4.		$A + 1 = 1$	$A.0 = 0$
5.	Double complement	$\overline{(\bar{A})} = A$	
6.	Commutative	$A + B = B + A$	$A.B = B.A$
7.	Associative	$A + (B + C) = (A + B) + C$	$A.(BC) = (AB)C$
8.	Distributive	$A(B + C) = AB + AC$	$A + BC = (A + B)(B + C)$
9.	De Morgan	$\overline{A + B} = \bar{A}.\bar{B}$	$\overline{A.B} = (\bar{A} + \bar{B})$
10.	Absorption	$A(A + B) = A$	$A + A.B = A$

Any Boolean function can be represented in a truth table.

If the number of binary variables is n then number of rows in the table is 2^n

Minimization of Boolean expressions u**sing algebraic method:**

Boolean expression can be simplified by using laws and theorems of Boolean algebra.

Sum of Product and Product of Sum

Logical functions are expressed in terms of logical variables.Two forms:

Sum of product (SOP)

Product of sum (POS)

Min Term: A product term containing all K variables is either complemented or complemented form

Max Term: sum term containing all variables is either complemented or complemented or complemented form.

Canonical SOP expression. It is the sum of all min terms derived from the rows of truth table.

Canonical POS Expression: Logical product of all max terms derived from the rows of truth table.

Prime implicant: It is the product term that cannot be simplified further by combination with other terms.

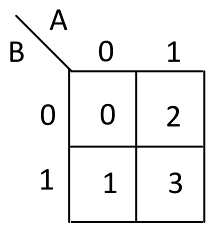
A problem has been discussed in video that how to derive SOP & POS expressions form truth table.

Karnaugh MAP (K-Map):

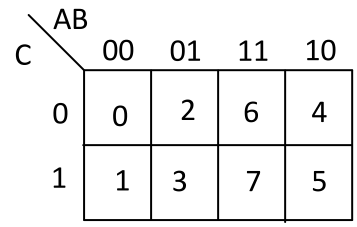
The simplification of switching functions using Boolean laws and theorems becomes complex with increase in number of variables and terms.

K-Map provide systematic method for simplifying and manipulating switching expressions. K-Map is modified form of truth table. N variable K-Map has 2^n cells entries of truth table can be entered in a K-Map. These maps can be drawn for 2, 3 or 4 variables e.tc.

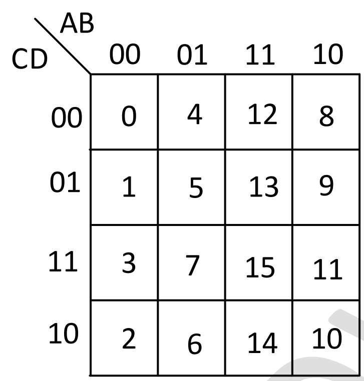
There is Gray code ordering of K-map rows and columns (adjacent row or columns differ by only 1 bit position).



(a) two variable



(b) three variable



(c) four variable

Simplification procedure:

- Construct K-Map enter 1's in the cells for SOP (0's for POS Expression)
- Group the adjacent 1's in two (pair), four (quad) or eight (octel) etc.
- The variables that remain same in all the cells of the group appear in the terms corresponding to the group or we can say that variables that change get eliminated.

Don't Care Combinations: In some digital systems, some input combinations never occur, such conditions are called don't care combinations. But these combinations can be plotted on the map to provide further simplification.

An example is shown in video.

Quine Mc. Clusky or tabular method of Minimization:

K-map is very effective tool for minimization upto 4 variables. Quine Mc. Clusky method also called tabular method is most suitable for more than 4 variables and above.

You can refer to any standard book on digital electronics.