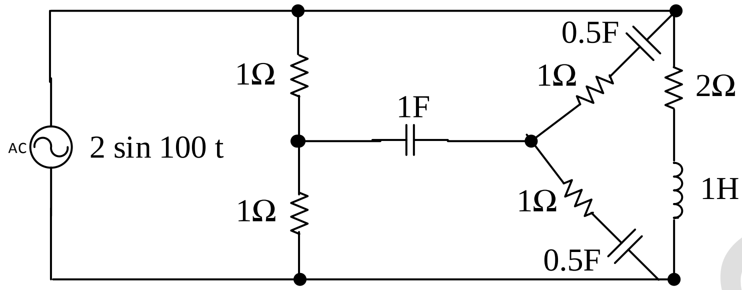
Sinusoidal Steady State-MCQs

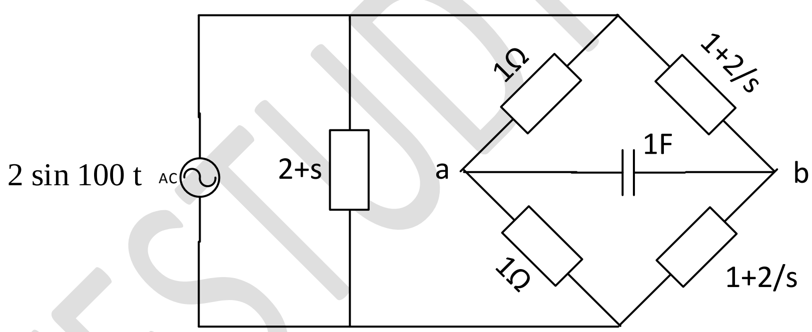
1. The value of current through the 1 Farad capacitor of figure is



(a) zero
(b) one
(c) two
(d) three

[GATE 1987: 2 Marks]

Sol. **The given circuit is a bridge circuit and can be redrawn as**



The product of the opposite arms are equal

$\boldsymbol{V_a = V_b}$

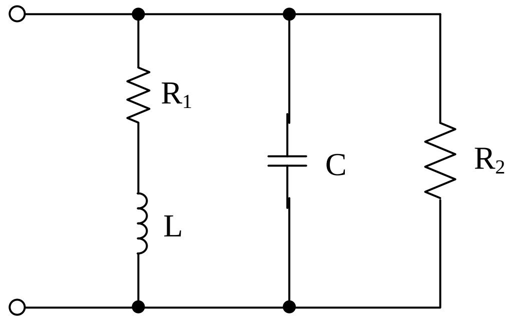
$$\boldsymbol{V_a = \frac{2\sin 100t}{2} \times 1 = \sin 100t}$$

$$\boldsymbol{V_b = \frac{2\sin 100t}{2(1+\frac{2}{s})} \times \left(1+\frac{2}{s}\right) = sin100t}$$

The current through 1F capacitor is zero

Option (a)

2. The half – power bandwidth of the resonant circuit of figure can be increased by:



(a) increasing R_1
(b) decreasing R_1
(c) increasing R_2
(d) decreasing R_2

[GATE 1989: 2 Marks]

Sol. **Bandwidth of the circuit** $\propto \frac{1}{Q}$

For increasing bandwidth, Q is to be decreased

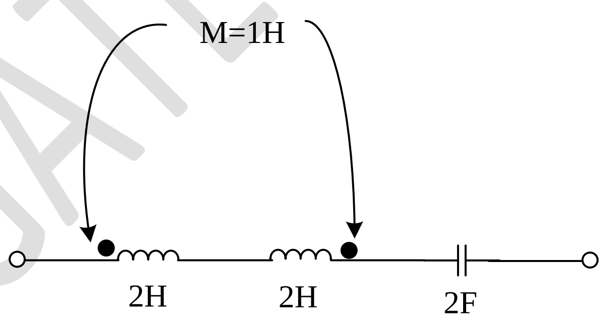
$$Q = \frac{\omega L}{R_1} = \frac{1}{\omega C R_1}$$

If $R_1 \rightarrow 0$, $R_2 \rightarrow \infty$, the circuit has only L & C elements and has high selectivity.

$R_1 \uparrow, R_2 \downarrow, Q \downarrow BW \uparrow$

Option (a) & (d)

3. The resonant frequency of the series circuit shown in figure is



(a) $\frac{1}{4\pi\sqrt{3}} H_Z$
(b) $\frac{1}{4\pi} H_Z$
(c) $\frac{1}{2\pi\sqrt{10}} H_Z$
(d) $\frac{1}{4\pi\sqrt{2}} H_Z$

[GATE 1990: 2 Marks]

Sol. **The coils are connected in a series opposing way. As the current is entering the dot of coil L_1 and leaving the dot of coil L_2**

$$Leq = L_1 + L_2 - 2M$$

$$= 2 + 2 - 2$$

$$= 2H$$

At resonance $X_L = X_C$

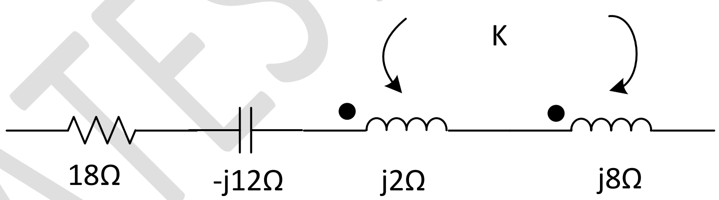
$$\omega L_{eq} = \frac{1}{\omega C}$$

$$\omega = \frac{1}{\sqrt{L_{eq}C}} = \frac{1}{\sqrt{2 \times 2}} = \frac{1}{2}\ rad/sec$$

$$f = \frac{1}{4\pi} Hz$$

Option (b)

4. In the series circuit shown in figure, for series resonance, the value of the coupling coefficient k will be



(a) 0.25
(b) 0.5
(c) 0.999
(d) 1.0

[GATE 1993: 1 Mark]

Sol. **The coils are connected in series additive manner as the current is entering the dot in both coils**

$$L_{eq} = L_1 + L_2 + 2M$$

$$M = K\sqrt{L_1 L_2} = K\sqrt{j2.j8} = Kj4$$

At resonance $|X_L| = |X_C|$

$$X_L = X_{L1} + X_{L2} + X_M$$

$$= j2 + j8 + 2Kj4$$

$$j10 + 2Kj4$$

At resonance $X_L = X_C$

$$j10 + 2Kj4 = j12$$

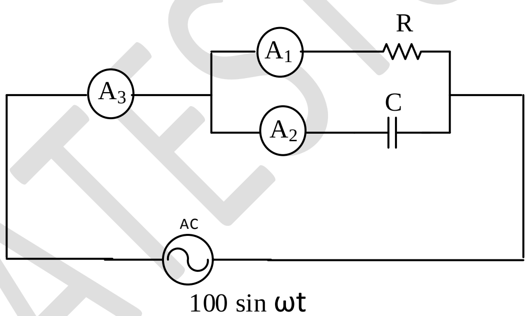
$$2Kj.4 = j.2$$

$$jK.8 = j.2$$

$$K = \frac{2}{8} = \frac{1}{4} = 0.25$$

Option (a)

5. In figure, A_1, A_2, and A_3 are ideal ammeters. If A_1 reads 5A, A_2 reads 12A, then A_3 should read



(a) 7 A
(b) 12 A
(c) 13 A
(d) 17 A

[GATE 1993: 2 Marks]

Sol. **Since the source is a.c.**

$$I_{3(rms)} = \sqrt{I_1^2(rms) + I_2^2(rms)}$$

$$= \sqrt{5^2 + 12^2}$$

$$= \sqrt{169} = 13A$$

Option (c)

6. A series LCR circuit consisting of $R = 10\ \Omega, |X_L| = 20\ \Omega\ and\ |X_C| = 20\ \Omega$ is connected across an a.c. supply of 200 V rms. The rms voltage across the capacitor is

(a) $200\angle -90^0\ V$ (c) $400\angle +90^0\ V$

(b) $200\angle +90^0\ V$ (d) $400\angle -90^0\ V$

[GATE 1994: 1 Mark]

Soln. **For a series LCR circuit**

$$\boldsymbol{Z = R + jX_L - jX_C}$$

$\boldsymbol{since\ X_L = X_C, \quad Z = R}$

$\boldsymbol{I = \frac{V}{R} = \frac{200}{10} = 20A}$

The voltage across the capacitor$= \boldsymbol{I.(-jX_C)}$

$= \boldsymbol{20.(-j20)}$

$= \boldsymbol{-400j}$

$= \boldsymbol{400\angle -90^0 V}$

Option (d)

7. A DC voltage source is connected across a series R-L-C circuit. Under steady state conditions, the applied DC voltage drops entirely across the

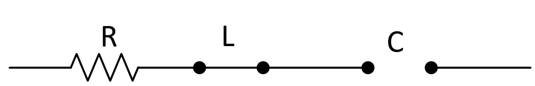
(a) R only (c) C only

(b) L only (d) R and L combination

[GATE 1995: 1 Mark]

Soln. **Under steady state condition, inductor behaves as a short circuit and capacitor as an open circuit.**

The applied voltage drops entirely across the capacitor



Option (C)

8. Consider a DC voltage source connected to a series R-C circuit. When the steady state reaches, the ratio of the energy stored in the capacitor to the total energy supplied by the voltage source, is equal to

(a) 0.362 (c) 0.632

(b) 0.500 (d) 1.000

[GATE 1995: 1 Mark]

Soln. **The total energy supplied by the source**

$$W_S = VI.t$$

$$= V\int_0^t I dt = Vq$$

$$= VCV$$

$$= CV^2$$

Voltage across the uncharged capacitor

$$V_C(t) = V_S\left(1 - e^{-t/RC}\right)$$

Power in the capacitor $= V_S\left(1 - e^{-t/RC}\right) C\frac{d}{dt}V_S\left(1 - e^{-t/RC}\right)$

$$= \frac{V_S^2}{R}\left(e^{-t/RC} - e^{-2t/RC}\right)$$

Energy stored in the capacitor at steady state

$$\frac{V_S^2}{R}\int_0^\infty e^{-t/RC}\,dt - \frac{V_S^2}{R}\int_0^\infty e^{-2t/RC}\,dt$$

$$= \frac{V_S^2}{R}\left[RC - \frac{RC}{2}\right]$$

$$= \frac{1}{2}CV_S^2$$

$$\frac{energy\ stored\ in\ the capacitor}{energy\ supplied\ by\ the\ source} = 0.5$$

Option (b)

9. The current, i(t), through a 10-Ω resistor in series with an inductance, is given by i(t) = 3 + 4 sin (100 t + 45^0) + 4 sin (300 t + 60^0) amperes.
The RMS value of the current and the power dissipated in the circuit are:

(a) $\sqrt{41}$A, 410 W, respectively
(b) $\sqrt{35}$ A, 350 W, respectively
(c) 5 A, 250 W, respectively
(d) 11 A, 1210 W respectively

[GATE 1995: 1 Mark]

Soln. $\boldsymbol{i(t) = 3 + 4\sin(100t + 45^0) + 4\sin(300t + 60^0)}$

$$\boldsymbol{I_{rms} = \sqrt{3^2 + \left(\frac{4}{\sqrt{2}}\right)^2 + \left(\frac{4}{\sqrt{2}}\right)^2} = 5\ amps}$$

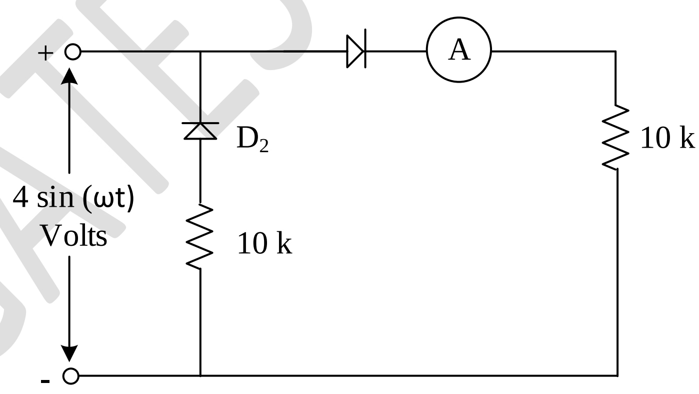
Power dissipated $\boldsymbol{= I_{rms}^2 R}$

$$= (\mathbf{5})^2 \times \mathbf{10}$$

$$= \mathbf{250}\ \boldsymbol{watts}$$

Option (c)

10. In the circuit of Figure assume that the diodes are ideal and the meter is an average indicating ammeter. The ammeter will read



(a) 0.4 $\sqrt{2}$ mA
(b) 0.4 mA
(c) $\frac{0.8}{\pi} mA$
(d) $\frac{0.4}{\pi} mA$

[GATE 1996: 1 Mark]

Soln. **For positive half cycle of input D_1 is conducting and D_2 is off**

The current through ammeter is the average value of positive half sine wave i.e.

$$I_{avg} = \frac{V_m}{\pi} \times \frac{1}{R} = \frac{4}{\pi \times 10 \times 10^3} = \frac{0.4}{\pi} mA$$

Option (d)

11. A series RLC circuit has a resonance frequency of 1 KHz and a quality factor Q = 100. If each of R, L and C is doubled from its original value, the new Q of the circuit is

(a) 25
(b) 50
(c) 100
(d) 200

[GATE 2003: 1 Mark]

Soln.

$$Q = \frac{\omega L}{R} = 2\pi f o \frac{L}{R}$$

$$= \frac{2\pi}{2\pi\sqrt{LC}} \frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

When R, L and C are doubled $Q' = \frac{Q}{2}$

Option (b)

12. An input voltage $v(t) = 10\sqrt{2}\cos(t + 10^0) + 10\sqrt{5}\cos(2t + 10^0)$ V is applied to a series combinationof R = 1 Ω and an inductance L = 1H. The resulting steady state current i(t) in ampere is

(a) $10\cos(t + 55^0) + 10\cos(2t + 10^0 + tan^{-1}2)$

(b) $10\cos(t + 55^0) + 10\sqrt{\frac{3}{2}}\cos(2t + 55)$

(c) $10\cos(t - 35^0) + 10\cos(2t + 10^0 - tan^{-1}2)$

(d) $10\cos(t-35^0)+10\sqrt{\frac{3}{2}}\cos(2t-35)$

[GATE 2003: 2 Marks]

Soln. $\boldsymbol{V(t)=10\sqrt{2}\cos(t+10^0)+10\sqrt{5}\ \cos(2t+10^0)}$

$$\boldsymbol{V_{(t)}=10\sqrt{2}\cos(t+10^0)+10\sqrt{5}\cos(2t+10^0)}$$

$$\boldsymbol{\omega_1=1,\omega_2=2}$$

Steady state current $\boldsymbol{i(t)=\ i_1(t)+i_2(t)}$

$$\boldsymbol{=\frac{10\sqrt{2}\cos(t+10^0)}{R+j\omega_1L}+\frac{10\sqrt{5}\cos(2t+10^0)}{R+j\omega_1L}}$$

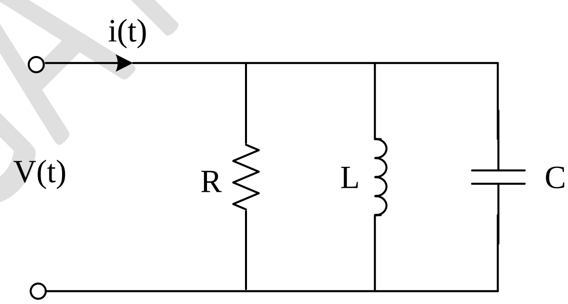
$$\boldsymbol{=\frac{10\sqrt{2}\cos(t+10^0)}{1+j}+\frac{10\sqrt{5}\cos(2t+10^0)}{1+2j}}$$

$$\boldsymbol{=\frac{10\sqrt{2}\cos(t+10^0)}{\sqrt{2}\angle 45^0}+\frac{10\sqrt{5}\cos(2t+10^0)}{\sqrt{5}\angle\tan^{-1}2}}$$

$$\boldsymbol{=10\cos(t-35^0)+10\cos(2t+10^2-\tan^{-1}2)}$$

Option (C)

13. The circuit shown in the figure, with $R=\frac{1}{3}\Omega, L=\frac{1}{4}H, C=3$ F has input voltage V(t) = Sin2t. The resulting current i(t) is



(a) $5\sin(2t+53.1^0)$

(b) $5\sin(2t-53.1^0)$

(c) $25\sin(2t+53.1^0)$

(d) $25\sin(2t-53.1^0)$

[GATE 2003: 1 Mark]

Soln. **Admittance**

$$Y = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

$$= 3 + \frac{4}{j2} + j2 \times 3$$

$$= 3 - j2 + j6$$

$$= 3 + j4$$

$$i(t) = V(t).y$$

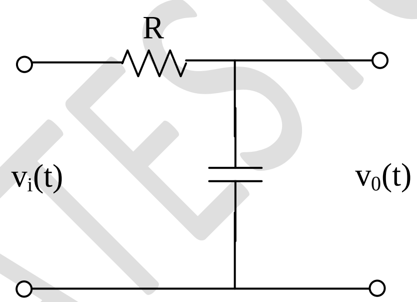
$$= (3 + 4j)\sin 2t$$

$$= 5 \sin 2t \angle \tan^{-1}(4/3)$$

$$= 5 \sin(2t + 53.1^0)$$

Option (a)

14. For the circuit shown in the figure, the time constant RC = 1 ms. The input voltage is $V_i(t) = \sqrt{2} \sin 10^3 t$. The output voltage V_0 (t) is equal to



(a) $\sin(10^3 t - 45^0)$

(b) $\sin(10^3 t + 45^0)$

(c) $\sin(10^3 t - 53^0)$

(d) $\sin(10^3 t + 53^0)$

[GATE 2004: 1 Mark]

Soln.

$$V_1(t) = \sqrt{2} \sin 10^3 t$$

$$\omega = 10^3\ radians/sec$$

$$RC = 1m\ sec$$

$$V_0(t) = \frac{V_1(t)}{\left(R + \frac{1}{j\omega c}\right)} \times \frac{1}{j\omega c}$$

$$= \frac{V_1(t)}{j\omega RC + 1} = \frac{\sqrt{2}\sin 10^3 t}{j10^3 \times 10^{-3} + 1}$$

$$= \frac{\sqrt{2}\sin 10^3 t}{1 + j}$$

$$= \sin(10^3 t - 45^0)$$

Option (a)

15. Consider the following statements S_1 and S_2

S_1: At the resonant frequency the impedance of a series R-L-C circuit is zero

S_2: In a parallel G-L-C circuit, increasing the conductance G results in increase in its Q factor.

Which one of the following is correct?

(a) S_1 is FALSE and S_2 is TRUE
(b) Both S_1 and S_2 are TRUE
(c) S_1 is TRUE and S_2 is FALSE
(d) Both S_1 and S_2 are FALSE

[GATE 2004: 2 Marks]

Soln. **The impedance of a series RLC circuit at resonant frequency is**

$$Z = R + j\left(\omega L - \frac{1}{\omega.C}\right)$$

$$\omega L = \frac{1}{\omega.C} \quad \textbf{At resonance}$$

So, Z = R purely resistive

In a parallel RLC circuit,

3dB bandwidth$\omega_2 - \omega_1 = \frac{1}{RC}$

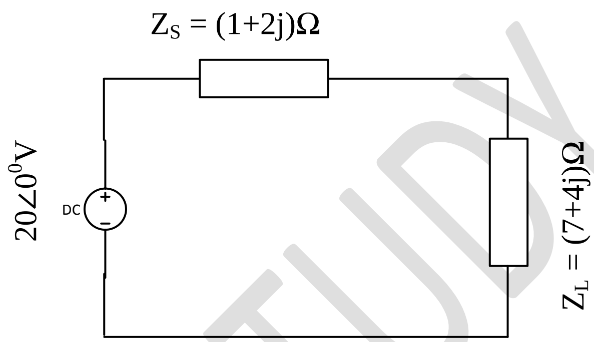
$$Q = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{\omega_r}{1/_{RC}} = \omega_R RC$$

ω_r is the resonant frequency.

As conductance $G \uparrow R \downarrow Q \downarrow$ (As conductance G increases R reduces so Q reduces)

Option (d)

16. An AC source of RMS voltage 20 V with internal impedance $Z_s = (1+2j)\Omega$ feeds a load of impedance $Z_L = (7+4j)\Omega$ in the figure below. The reactive power consumed by the load is



(a) 8 VAR
(b) 16 VAR
(c) 28 VAR
(d) 32 VAR

[GATE 2009: 2 Marks]

Soln. **The current I in the given circuit is**

$$I = \frac{20}{8+6j} = \frac{10}{4+3j}$$

$$= \frac{10}{\sqrt{4^2+3^2}\angle \tan^{-1} 3/4}$$

$$= \frac{10}{5\angle 36.87} = 2\angle -36.87^0$$

Reactive power $= \boldsymbol{I^2 X_L}$

$= (\mathbf{2})^2 \times \mathbf{4}$

= 16 VAR

Option (b)