1. The value of current through the 1 Farad capacitor of figure is

\[ \text{AC } 2 \sin 100 t \]

(a) zero  
(b) one  
(c) two  
(d) three  

[GATE 1987: 2 Marks]

Sol. The given circuit is a bridge circuit and can be redrawn as

\[ V_a = V_b \]

\[ V_a = \frac{2 \sin 100t}{2} \times 1 = \sin 100t \]

\[ V_b = \frac{2 \sin 100t}{2(1 + \frac{2}{s})} \times \left(1 + \frac{2}{s}\right) = sin100t \]

The current through 1F capacitor is zero

Option (a)
2. The half – power bandwidth of the resonant circuit of figure can be increased by:

(a) increasing $R_1$
(b) decreasing $R_1$
(c) increasing $R_2$
(d) decreasing $R_2$

[GATE 1989: 2 Marks]

Sol. Bandwidth of the circuit $\propto \frac{1}{Q}$

For increasing bandwidth, $Q$ is to be decreased

$$Q = \frac{\omega L}{R_1} = \frac{1}{\omega CR_1}$$

If $R_1 \to 0, R_2 \to \infty$, the circuit has only L & C elements and has high selectivity.

$R_1 \uparrow, R_2 \downarrow, Q \downarrow BW \uparrow$

Option (a) & (d)

3. The resonant frequency of the series circuit shown in figure is

(a) $\frac{1}{4\pi\sqrt{3}}Hz$
(b) $\frac{1}{4\pi}Hz$
(c) $\frac{1}{2\pi\sqrt{10}}Hz$
(d) $\frac{1}{4\pi\sqrt{2}}Hz$

[GATE 1990: 2 Marks]
Sol. The coils are connected in a series opposing way. As the current is entering the dot of coil $L_1$ and leaving the dot of coil $L_2$

$$Leq = L_1 + L_2 - 2M$$

$$= 2 + 2 - 2$$

$$= 2H$$

At resonance $X_L = X_C$

$$\omega Leq = \frac{1}{\omega C}$$

$$\omega = \frac{1}{\sqrt{Leq C}} = \frac{1}{\sqrt{2 \times 2}} = \frac{1}{2} \text{ rad/sec}$$

$$f = \frac{1}{4\pi} Hz$$

Option (b)

4. In the series circuit shown in figure, for series resonance, the value of the coupling coefficient $k$ will be

![Series Circuit Diagram]

(a) 0.25

(b) 0.5

(c) 0.999

(d) 1.0

[GATE 1993: 1 Mark]

Sol. The coils are connected in series additive manner as the current is entering the dot in both coils

$$Leq = L_1 + L_2 + 2M$$

$$M = K\sqrt{L_1 L_2} = K\sqrt{j2 \cdot j8} = Kj4$$

At resonance $|X_L| = |X_C|$
\[ X_L = X_{L1} + X_{L2} + X_M \]
\[ = j2 + j8 + 2Kj4 \]
\[ j10 + 2Kj4 \]

At resonance \( X_L = X_C \)
\[ j10 + 2Kj4 = j12 \]
\[ 2K.j.4 = j.2 \]
\[ jK.8 = j.2 \]

\[ K = \frac{2}{8} = \frac{1}{4} = 0.25 \]

Option (a)

5. In figure, \( A_1, A_2, \) and \( A_3 \) are ideal ammeters. If \( A_1 \) reads 5A, \( A_2 \) reads 12A, then \( A_3 \) should read

(a) 7 A  
(b) 12 A  
(c) 13 A  
(d) 17 A

[\text{GATE 1993: 2 Marks}]

Sol. Since the source is a.c.

\[ I_{3(rms)} = \sqrt{I_{1(rms)}^2 + I_{2(rms)}^2} \]
\[ = \sqrt{5^2 + 12^2} \]
\[ = \sqrt{169} = 13A \]
Option (c)

6. A series LCR circuit consisting of \( R = 10 \Omega, |X_L| = 20 \Omega \) and \( |X_C| = 20 \Omega \) is connected across an a.c. supply of 200 V rms. The rms voltage across the capacitor is

(a) \( 200 \angle -90^0 \) V
(b) \( 200 \angle +90^0 \) V
(c) \( 400 \angle +90^0 \) V
(d) \( 400 \angle -90^0 \) V

[Gate 1994: 1 Mark]

Soln. For a series LCR circuit

\[ Z = R + jX_L - jX_C \]

Since \( X_L = X_C \), \( Z = R \)

\[ I = \frac{V}{R} = \frac{200}{10} = 20A \]

The voltage across the capacitor = \( I.(-jX_C) \)

\[ = 20.(-j20) \]

\[ = -400j \]

\[ = 400 \angle -90^0 V \]

Option (d)

7. A DC voltage source is connected across a series R-L-C circuit. Under steady state conditions, the applied DC voltage drops entirely across the

(a) R only
(b) L only
(c) C only
(d) R and L combination

[Gate 1995: 1 Mark]

Soln. Under steady state condition, inductor behaves as a short circuit and capacitor as an open circuit.

The applied voltage drops entirely across the capacitor

Option (C)
8. Consider a DC voltage source connected to a series R-C circuit. When the steady state reaches, the ratio of the energy stored in the capacitor to the total energy supplied by the voltage source, is equal to

(a) 0.362  
(b) 0.500  
(c) 0.632  
(d) 1.000

[GATE 1995: 1 Mark]

Soln. The total energy supplied by the source

\[ W_s = \int I \, dt = Vq \]

\[ = VCV \]

\[ = CV^2 \]

Voltage across the uncharged capacitor

\[ V_C(t) = V_s \left( 1 - e^{-t/RC} \right) \]

Power in the capacitor

\[ = V_S \left( 1 - e^{-t/RC} \right) C \frac{d}{dt} V_S \left( 1 - e^{-t/RC} \right) \]

\[ = \frac{V_S^2}{R} \left( e^{-t/RC} - e^{-2t/RC} \right) \]

Energy stored in the capacitor at steady state

\[ = \frac{V_S^2}{R} \int_0^\infty e^{-t/RC} \, dt - \frac{V_S^2}{R} \int_0^\infty e^{-2t/RC} \, dt \]

\[ = \frac{V_S^2}{R} \left[ RC - \frac{RC}{2} \right] \]

\[ = \frac{1}{2} CV_S^2 \]

\[ \frac{\text{energy stored in the capacitor}}{\text{energy supplied by the source}} = 0.5 \]
9. The current, \( i(t) \), through a 10-\( \Omega \) resistor in series with an inductance, is given by
\[
i(t) = 3 + 4 \sin(100t + 45^0) + 4 \sin(300t + 60^0) \text{ amperes.}
\]
The RMS value of the current and the power dissipated in the circuit are:

(a) \( \sqrt{41} \text{A, } 410 \text{ W, respectively} \)
(b) \( \sqrt{35} \text{ A, } 350 \text{ W, respectively} \)
(c) 5 A, 250 W, respectively
(d) 11 A, 1210 W respectively

\[ \text{[GATE 1995: 1 Mark]} \]

**Soln.**
\[
i(t) = 3 + 4 \sin(100t + 45^0) + 4 \sin(300t + 60^0)
\]
\[
I_{rms} = \sqrt{3^2 + \left( \frac{4}{\sqrt{2}} \right)^2 + \left( \frac{4}{\sqrt{2}} \right)^2} = 5 \text{ amps}
\]
\[
\text{Power dissipated} = I_{rms}^2R
\]
\[
= (5)^2 \times 10
\]
\[
= 250 \text{ watts}
\]

Option (c)

10. In the circuit of Figure assume that the diodes are ideal and the meter is an average indicating ammeter. The ammeter will read

\[ \text{[GATE 1996: 1 Mark]} \]

(a) \( 0.4 \sqrt{2} \text{ mA} \)
(b) 0.4 mA
(c) \( \frac{0.8}{\pi} \text{ mA} \)
(d) \( \frac{0.4}{\pi} \text{ mA} \)
For positive half cycle of input D₁ is conducting and D₂ is off

The current through ammeter is the average value of positive half sine wave i.e.

\[ I_{avg} = \frac{V_m}{\pi} \times \frac{1}{R} = \frac{4}{\pi \times 10 \times 10^3} = \frac{0.4}{\pi} mA \]

Option (d)

11. A series RLC circuit has a resonance frequency of 1 KHz and a quality factor \( Q = 100 \). If each of R, L and C is doubled from its original value, the new \( Q \) of the circuit is

(a) 25  
(b) 50  
(c) 100  
(d) 200

\[ Q = \frac{\omega L}{R} = \frac{2\pi f_0 L}{R} = \frac{2\pi}{2\pi} \sqrt{\frac{L}{C R}} = \frac{1}{R} \sqrt{\frac{L}{C}} \]

When R, L and C are doubled \( Q' = \frac{Q}{2} \)

Option (b)

12. An input voltage \( v(t) = 10\sqrt{2} \cos(t + 10^0) + 10\sqrt{5} \cos(2t + 10^0) \) V is applied to a series combination of R = 1 Ω and an inductance L = 1H. The resulting steady state current \( i(t) \) in ampere is

(a) \( 10 \cos(t + 55^0) + 10 \cos(2t + 10^0 + \tan^{-1}2) \)

(b) \( 10 \cos(t + 55^0) + 10 \sqrt{\frac{3}{2}} \cos(2t + 55) \)

(c) \( 10 \cos(t - 35^0) + 10 \cos(2t + 10^0 - \tan^{-1}2) \)
(d) $10 \cos(t - 35^0) + 10 \sqrt{2} \cos(2t - 35^0)$

\[ V(t) = 10 \sqrt{2} \cos(t + 10^0) + 10 \sqrt{5} \cos(2t + 10^0) \]
\[ V(t) = 10 \sqrt{2} \cos(t + 10^0) + 10 \sqrt{5} \cos(2t + 10^0) \]
\[ \omega_1 = 1, \omega_2 = 2 \]

Steady state current $i(t) = i_1(t) + i_2(t)$

\[
\begin{align*}
&= \frac{10 \sqrt{2} \cos(t + 10^0)}{R + j \omega_1 L} + \frac{10 \sqrt{5} \cos(2t + 10^0)}{R + j \omega_1 L} \\
&= \frac{10 \sqrt{2} \cos(t + 10^0)}{1 + j} + \frac{10 \sqrt{5} \cos(2t + 10^0)}{1 + 2j} \\
&= \frac{10 \sqrt{2} \cos(t + 10^0)}{\sqrt{2} \angle 45^0} + \frac{10 \sqrt{5} \cos(2t + 10^0)}{\sqrt{5} \angle \tan^{-1} 2} \\
&= 10 \cos(t - 35^0) + 10 \cos(2t + 10^2 - \tan^{-1} 2)
\end{align*}
\]

Option (C)

13. The circuit shown in the figure, with $R = \frac{1}{3} \Omega, L = \frac{1}{4} H, C = 3 \ F$ has input voltage $V(t) = \sin 2t$. The resulting current $i(t)$ is

(a) $5 \sin(2t + 53.1^0)$
(b) $5 \sin(2t - 53.1^0)$
(c) $25 \sin(2t + 53.1^0)$
(d) $25 \sin(2t - 53.1^0)$

\[ \text{[GATE 2003: 1 Mark]} \]
Soln. Admittance

\[ Y = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C \]

\[ = 3 + \frac{4}{j2} + j2 \times 3 \]

\[ = 3 - j2 + j6 \]

\[ = 3 + j4 \]

\[ i(t) = V(t) \cdot y \]

\[ = (3 + 4j) \sin 2t \]

\[ = 5 \sin 2t \angle \tan^{-1}(4/3) \]

\[ = 5 \sin(2t + 53.1^0) \]

Option (a)

14. For the circuit shown in the figure, the time constant \( RC = 1 \) ms. The input voltage is \( V_i(t) = \sqrt{2} \sin 10^3t \). The output voltage \( V_0(t) \) is equal to

\[ \text{Option (a)} \]

\[ \text{Option (b)} \]

\[ \text{Option (c)} \]

\[ \text{Option (d)} \]

[\text{GATE 2004: 1 Mark}]

Soln. \( V_1(t) = \sqrt{2} \sin 10^3t \)

\( \omega = 10^3 \text{ radians/sec} \)

\( RC = 1 \text{ m sec} \)
\[ V_0(t) = \frac{V_1(t)}{R + \frac{1}{j\omega C}} \times \frac{1}{j\omega C} \]

\[ = \frac{V_1(t)}{j\omega RC + 1} = \frac{\sqrt{2} \sin 10^3 t}{10^3 \times 10^{-3} + 1} \]

\[ = \frac{\sqrt{2} \sin 10^3 t}{1 + j} \]

\[ = \sin(10^3 t - 45^\circ) \]

Option (a)

15. Consider the following statements S_1 and S_2

S_1: At the resonant frequency the impedance of a series R-L-C circuit is zero
S_2: In a parallel G-L-C circuit, increasing the conductance G results in increase in its Q factor.

Which one of the following is correct?

(a) S_1 is FALSE and S_2 is TRUE
(b) Both S_1 and S_2 are TRUE
(c) S_1 is TRUE and S_2 is FALSE
(d) Both S_1 and S_2 are FALSE

[SATE 2004: 2 Marks]

Soln. The impedance of a series RLC circuit at resonant frequency is

\[ Z = R + j \left( \omega L - \frac{1}{\omega C} \right) \]

\[ \omega L = \frac{1}{\omega C} \quad \text{At resonance} \]

So, \( Z = R \) purely resistive

In a parallel RLC circuit,

\[ 3\text{dB bandwidth} \omega_2 - \omega_1 = \frac{1}{RC} \]
\[ Q = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{\omega_r}{1/RC} = \omega R C \]

\( \omega_r \) is the resonant frequency.

As conductance \( G \uparrow \) \( R \downarrow \) \( Q \downarrow \) (As conductance \( G \) increases \( R \) reduces so \( Q \) reduces)

Option (d)

16. An AC source of RMS voltage 20 V with internal impedance \( Z_s = (1 + 2j) \Omega \) feeds a load of impedance \( Z_L = (7 + 4j) \Omega \) in the figure below. The reactive power consumed by the load is

\( Z_s = (1+2j)\Omega \)

[DC 20 00V
\( Z_s = (1+2j)\Omega \)
\( Z_L = (7+4j)\Omega \)

(a) 8 VAR  
(b) 16 VAR  
(c) 28 VAR  
(d) 32 VAR

[GATE 2009: 2 Marks]

Soln. The current \( I \) in the given circuit is

\[ I = \frac{20}{8 + 6j} = \frac{10}{4 + 3j} \]

\[ = \frac{10}{\sqrt{4^2 + 3^2} \angle \text{tan}^{-1} 3/4} \]

\[ = \frac{10}{5 \angle 36.87} = 2 \angle -36.87^0 \]
Reactive power $= I^2 X_L$

$= (2)^2 \times 4$

$= 16 \text{ VAR}$

Option (b)