Transient and Steady State Analysis

1. A 10 Ω resistor, a 1 H inductor and 1 μF capacitor are connected in parallel. The combination is driven by a unit step current. Under the steady state condition, the source current flows through.

(a) the resistor
(b) the inductor
(c) the capacitor only
(d) all the three elements

[GATE 1989: 2 Marks]

Soln. **Under steady state condition, the capacitor is open circuit and inductor short circuit. The source current flows through inductor.**

Option (a)

2. If the Laplace transform of the voltage across a capacitor of value of ½ F is

$$V_C(S) = \frac{S+1}{S^3+S^2+S+1}$$

The value of the current through the capacitor at t = 0^+ is

(a) 0 A
(b) 2 A
(c) (1/2) A
(d) 1 A

[GATE 1989: 2 Marks]

Soln. **Impedance of capacitor**

$$= \frac{1}{CS} = \frac{2}{S}$$

Current through the capacitor

$$= \frac{V_C(S)}{Z_C(S)}$$

$$= \frac{(S+1)\,S}{(S^3+S^2+S+1)^2} = \frac{S}{2\,(S^2+1)}$$

$$i(0_+) = \lim_{S\to\infty} S\ \ i_C(S)$$

$$= \lim_{S\to\infty} \frac{S^2}{2\,(S^2+1)} = \lim_{S\to\infty} \frac{1}{2\left(1+\frac{1}{S^2}\right)} = \frac{1}{2}$$

Option (c)

3. A ramp voltage, v(t) = 100 t volts, is applied to an RC differentiating circuit with R = 5 kΩ and C = 4 μF. The maximum output voltage is

(a) 0.2 volt
(b) 2.0 volts
(c) 10.0 volts
(d) 50.0 volts

[GATE 1994: 1 Mark]

Soln. The output of an RC differentiating circuit

$$= RC\frac{dv_1}{dt}$$

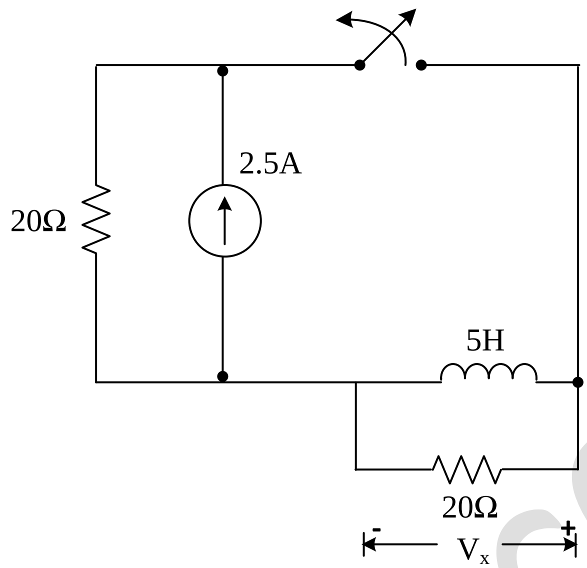
$$= 5 \times 10^3 \times 4 \times 10^{-6}\ \frac{d}{dt}(100t)$$

$$= 20 \times 10^{-3} \times 100$$

$$= 2\ votls$$

Option (b)

4. In the figure, the switch was closed for a long time before opening at t = 0. The voltage V_x at t = 0^+ is



(a) 25 V
(b) 50 V
(c) -50 V
(d) 0 V

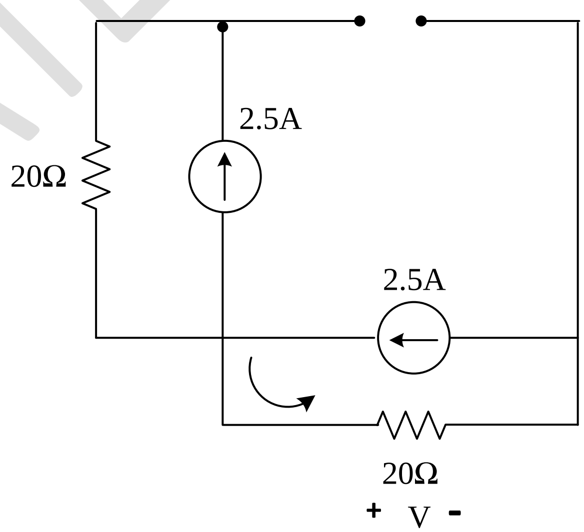
[GATE 2002 :1 Mark]

Soln. **When the switch was closed for a long time, the steady state is reached and inductor is short circuit.**

$$I_L(0_-) = 2.5\ A$$

The current cannot change instantaneously in a inductor $I_L(0_-) = I_L(0_+) = 2.5\ A$

At $t = 0_+$ inductor can be replaced by a current source of 2.5A. The equivalent circuit for the same is drawn.



$$V = 20 \times 2.5$$

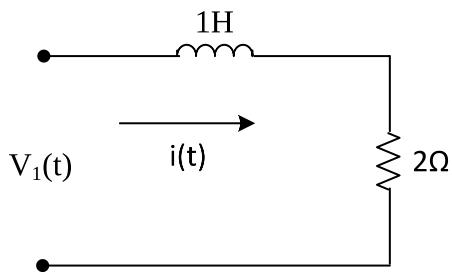
$$= 50\ V$$

$V_X = V$ **is of opposite polarity**

$$V_X = -50V$$

Option (c)

5. For the R-L circuit shown in the figure, the input voltage $V_i(t) = u(t)$. Plot the current i(t)



Soln.

$$I(S) = \frac{V_1(S)}{LS + 2} = \frac{1}{S(S+2)}$$

$$= \frac{1}{2}\left[\frac{1}{S} - \frac{1}{S+2}\right]$$

$$i(t) = \frac{1}{2} - \frac{1}{2}e^{-2t}$$

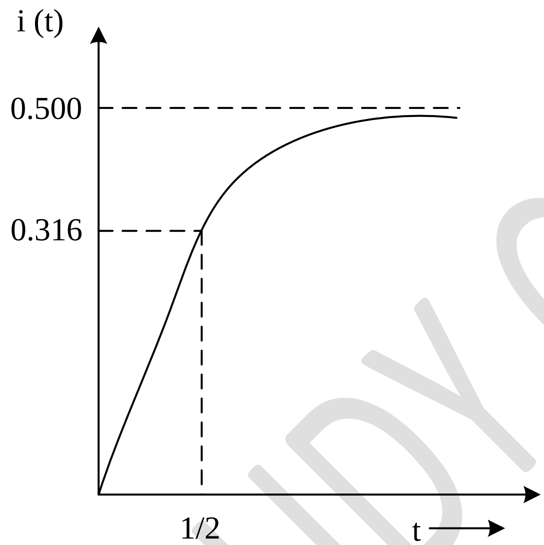
$$at\ t = 0, \quad i(0) = \frac{1}{2} - \frac{1}{2} = 0$$

$$at\ t = \infty, \quad i(\infty) = \frac{1}{2} = 0.5$$

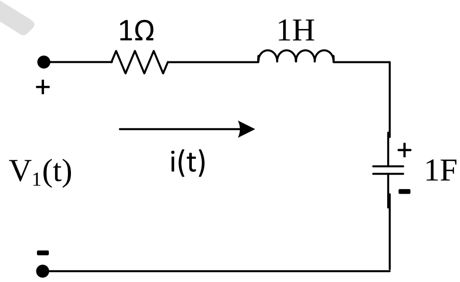
$$at\ t = \frac{1}{2}, \quad i\left(\frac{1}{2}\right) = \frac{1}{2}(1 - e^{-1})$$

$$= \frac{1}{2}(1 - 0.368) = \frac{0.632}{2}$$

$$= 0.316$$



6. The circuit shown in the figure has initial current $i_L(0^-) = 1$ A through the inductor and an initial voltage $V_C(0^-) = -1$ V across the capacitor. For input v(t) = u(t) find the laplace transform of the current i(t) for $t \geq 0$



[GATE 2004 :2 Marks]

Soln. **Writing KVL**

$$V(t) = Ri + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{t} i\,dt$$

Taking Laplace transform on both sides

$$V(S) = RI(S) + L[SI(S) - i(0_+)] + \frac{1}{C}\left[\frac{I(S)}{S} + \frac{q(0_-)}{S}\right]$$

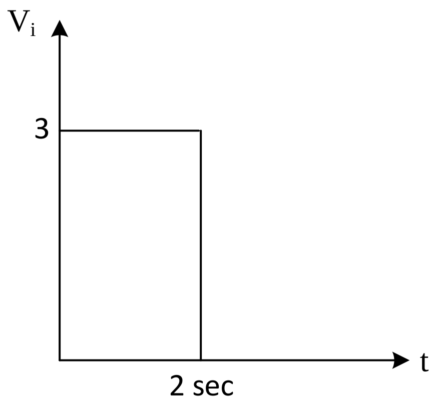
Where $i(0_+)$ is the initial current and $\frac{q(0_-)}{C}$ is the initial voltage of the capacitor.

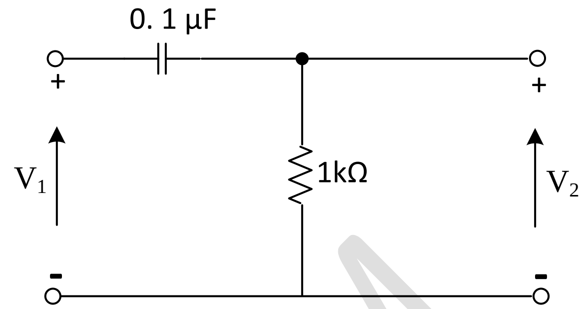
$$\frac{1}{S} = I(S) + SI(S) - 1 + \frac{I(S)}{S} - \frac{1}{S}$$

$$\frac{2}{S} + 1 = I(S)\left[1 + S + \frac{1}{S}\right]$$

$$I(S) = \frac{S+2}{S^2 + S + 1}$$

7. A square pulse of 3 volts amplitude is applied to C-R circuit shown in the figure. The capacitor is initially uncharged. The output voltage V_2 at time t = 2 sec is





(a) 3 V
(b) -3 V
(c) 4 V
(d) -4 V

[GATE : 2005 2 Marks]

Soln. **Time constant**

$$RC = 1 \times 10^{3} \times 0.1 \times 10^{-6}$$

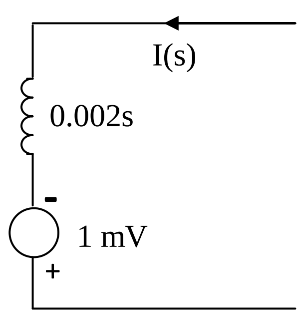
$$= 0.1 \times 10^{-3}\ sec$$

$$= 0.1\ m\ sec$$

Steady state will be reached in time ≥ 5 time constant (0.5 m sec). The capacitor gets charge to + 3 volts $V_2 = -3\ volts$

Option (b)

8. A 2 mH inductor with some initial current is in figure. Where s is the laplace transform variable. The value of initial current is.



(a) 0.5 A
(b) 2.0 A
(c) 1.0 A
(d) 0.0 A

[GATE : 2006 1 Mark]

Soln. Voltage across inductor L

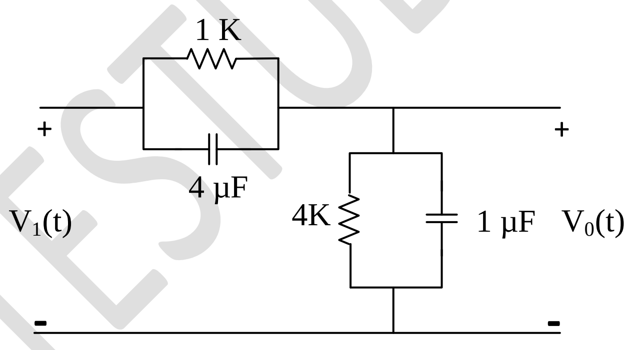
$$\boldsymbol{L = L\frac{di}{dt} = L[SI(S) - i(0_+)]}$$

$$\boldsymbol{L\, i(0_+) = 1\ mv}$$

$$\boldsymbol{i(0_+) = \frac{1\ mv}{2\ mH} = 0.5A}$$

Option (a)

9. In the figure shown below, assume that all the capacitors are initially uncharged. If V_i (t) = 10 u(t) Volts, V_0(t) is given by



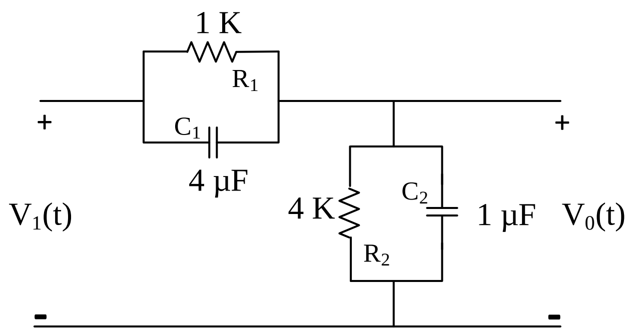
(a) $8e^{-t/0.004}$ Volts
(b) $8\,(1 - e^{-t/0.004})$ Volts
(c) 8u(t) Volts
(d) 8 Volts

[GATE : 2006 1 Mark]

Soln. Let

$$\boldsymbol{Z_1 = 1K \| 4\ \mu F = \frac{R_1}{R_1C_1S + 1}}$$

$$\boldsymbol{Z_1 = \frac{10^3}{4 \times 10^{-3}S + 1}}$$



$$\boldsymbol{Z_2 = 4K \| 1\mu F}$$

$$\boldsymbol{Z_2 = \frac{R_2}{R_2C_2S + 1} = 4\ Z_1}$$

$$\boldsymbol{V_0 t = \frac{Z_2}{Z_1 + Z_2} V_1(t)}$$

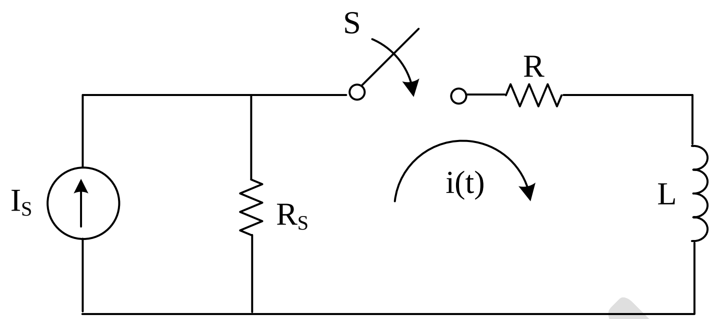
$$\boldsymbol{= \frac{4Z_1}{5Z_1} V_1 t = 0.8 V_1(t)}$$

$$\boldsymbol{= 0.8 \times 10u(t)}$$

$$\boldsymbol{= 8u(t)}$$

Option (c)

10. In the following circuit, the switch S is closed at t = 0. The rate of change of current $\frac{di}{dt}(0^+)$ *is given by*



(a) 0

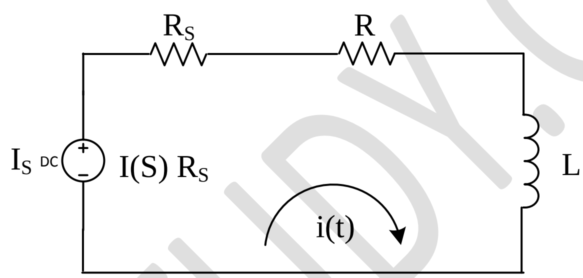
(b) $\frac{R_S I_S}{L}$

(c) $\frac{(R+R_S)I_S}{L}$

(d) ∞

[GATE : 2008 1 Mark]

Soln. Drawing the equivalent circuit as the switch is closed



$$\boldsymbol{I_S R_S = (R + R_S)i(t) + L\frac{di}{dt}}$$

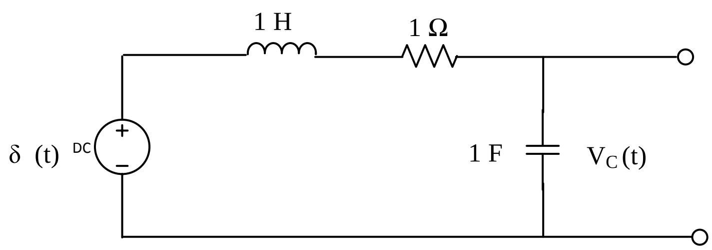
$$\boldsymbol{For\ t = 0^+, I_S R_S = (R + R_S)i(0_+) + L\frac{di}{dt}(0_+)}$$

Since current can not change instantaneously in a inductor so $\boldsymbol{i(0_+) = i(0_-) = 0}$

$$\boldsymbol{\frac{di}{dt}(0_+) = \frac{I_S R_S}{L}}$$

Option (b)

11. For t > 0, the voltage across the capacitor is:



(a) $\frac{1}{\sqrt{3}}\left(e^{-\frac{\sqrt{3}}{2}t} - e^{-\frac{1}{2}t}\right)$

(b) $e^{-\frac{1}{2}t}\left[cos\left(\frac{\sqrt{3}\,t}{2}\right) - \frac{1}{\sqrt{3}}sin\left(\frac{\sqrt{3}\,t}{2}\right)\right]$

(c) $\frac{2}{\sqrt{3}}e^{-\frac{1}{2}t}sin\left(\frac{\sqrt{3}\,t}{2}\right)$

(d) $\frac{2}{\sqrt{3}}e^{-\frac{1}{2}t}cos\left(\frac{\sqrt{3}\,t}{2}\right)$

[GATE: 2008 2 Marks]

Soln. Writing KVL

$$\boldsymbol{\delta(t) = Ri + L\frac{di}{dt} + \frac{1}{C}\int_0^t idt}$$

Taking laplace transform on both sides

$$\boldsymbol{1 = I(S)\left[R + LS + \frac{1}{CS}\right]}$$

$$\boldsymbol{I(S) = \frac{1}{\left(S + 1 + \frac{1}{S}\right)} = \frac{S}{(S^2 + S + 1)}}$$

$$\boldsymbol{V_C(S) = \frac{I(S)}{CS} = \frac{S}{(S^2 + S + 1)S}}$$

$$\boldsymbol{= \frac{1}{(S^2 + S + 1)}}$$

$$\boldsymbol{= \frac{1}{\left(S + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}}$$

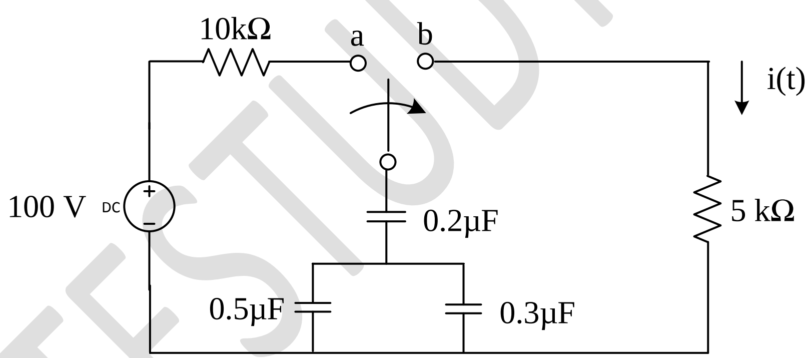
$$\boldsymbol{Laplace\ of\quad e^{-at}\sin \omega t = \frac{\omega}{(S + a)^2 + \omega^2}}$$

$$V_C(S) = \frac{\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}\left[\left(S+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2\right]}$$

$$V_C(t) = \frac{2}{\sqrt{3}} e^{-\frac{t}{2}} \sin\left(\frac{\sqrt{3}}{2} t\right)$$

Option(c)

12. The switch in the circuit shown was on position a for a long time, and is moved to position b at time t = 0. Find the current i(t) for t > 0



[GATE : 2009 2 Marks]

Soln. Total capacitance : C

$$\frac{1}{C} = \frac{1}{0.2} + \frac{1}{0.8}$$

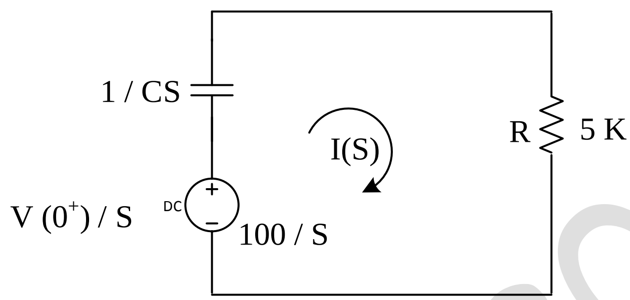
$$= 5 + \frac{10}{8}$$

$$= \frac{50}{8}$$

$$C = \frac{8}{50} = 0.16\mu f$$

The switch is in position a for a long a for a long time, the capacitor C is fully charged to 100 volts.

When switch is in position b, the equivalent circuit for the same is



$$I(S) = \frac{100/S}{R + \frac{1}{CS}}$$

$$= \frac{100}{S\left[R + \frac{1}{CS}\right]}$$

$$= \frac{100}{R\left[S + \frac{1}{RC}\right]}$$

$$i(t) = \frac{100}{R}\ e^{-t/RC}$$

$$= 20\ e^{-1250t} ma$$

$$= 20\ e^{-1250t} u(t)\ ma$$

13. The time domain behavior of an RL circuit is represented by

$$L\frac{di}{dt} + Ri = V_0\left(1 + Be^{-\frac{Rt}{L}}\ sin\ t\right)u(t)$$

For an initial current of $i(0) = \frac{V_0}{R}$ the steady state value of the current is given by

(a) $i(t) \to \frac{V_0}{R}$

(b) $i(t) \to \frac{2V_0}{R}$

(c) $i(t) \to \frac{V_0}{R}(1 + B)$

(d) $i(t) = \frac{2V_0}{R}(1 + B)$

[GATE : 2009 2 Marks]

Soln.

$$L\frac{di}{dt} + Ri = V_0\left(1 + Be^{-\frac{Rt}{L}}\ \sin t\right)u(t)$$

Taking Laplace transform

$$L[SI(S) - i(0_+)] + RI(S) = V_0\left[\frac{1}{S} + \frac{B}{\left(S + \frac{R}{L}\right)^2 + 1}\right]$$

$$I(S)[R + LS] - L\,i(0_+) = V_0\left[\frac{1}{S} + \frac{B}{\left(S + \frac{R}{L}\right)^2 + 1}\right]$$

$$I(S) = \frac{V_0}{LS + R}\left[\frac{1}{S} + \frac{B}{\left(S + \frac{R}{L}\right)^2 + 1}\right] + L\,i(0_+)$$

The steady sate value of current is

$$\lim_{t\to\infty} i(t) = \lim_{S\to 0} SI(S)$$

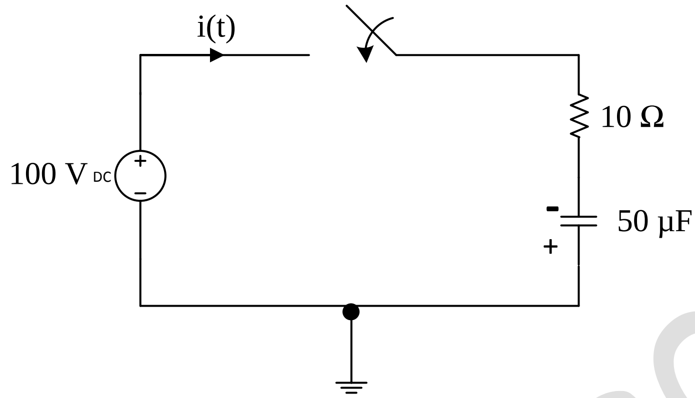
$$SI(S) = \frac{V_0}{LS + R}\left[1 + \frac{BS}{S^2 + \frac{R^2}{L^2} + 1}\right] + L\frac{V_0}{R}S$$

$$\lim_{S\to 0} SI(S) = \frac{V_0}{R}[1 + 0] + 0$$

$$\lim_{t\to\infty} i(t) = \frac{V_0}{R}$$

Option (a)

14. In the circuit shown below, the initial charge on the capacitor is 2.5 mC, with the voltage polarity as indicated. The switch is closed at time t = 0. The current i(t) at a time t after the switch is closed is



(a) $i(t) = 15\exp(-2 \times 10^3 t)A$

(b) $i(t) = 5\exp(-2 \times 10^3 t)A$

(c) $i(t) = 10\exp(-2 \times 10^3 t)A$

(d) $i(t) = -5\exp(-2 \times 10^3 t)A$

[GATE : 2011 2 Marks]

Soln. **Initial charge = 2.5 mc**

Initial voltage on capacitor

$$= \frac{2.5 \times 10^{-3}}{50 \times 10^{-6}} = 50V$$

Net voltage

$$= 100 + 50 = 150$$

$$i(t) = \frac{v}{R} e^{-t/RC}$$

Where

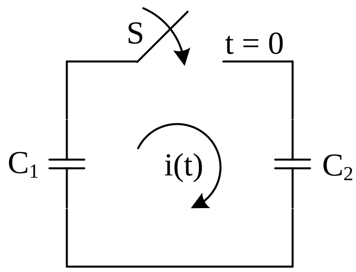
$$RC = 10 \times 50 \times 10^{-6} = 5 \times 10^{-4}\ sec$$

$$i(t) = \frac{150}{10}\ e^{-t/5\times10^{-4}}$$

$$= 15e^{(-2\times10^3\ t)}\ A$$

Option (a)

15. In the following figure C_1 and C_2 are ideal capacitors. C_1 had been charged to 12V before the ideal switch S is closed at t = 0. The current i(t) for all t is



(a) Zero
(b) A step function
(c) An exponentially decaying function
(d) An impulse function

[GATE : 2012 1 Mark]

Soln. **Since there is no resistance so time constant is zero. This means as the switch is closed, C_2 will get charge. Charging and discharge time constant is zero. Sudden change of voltage exists only if impulse of current passes through it.**

Option (d)