Amplitude Modulated Systems

(a) Modulation

1. In commercial TV transmission in India, picture and speech signals are modulated respectively

<table>
<thead>
<tr>
<th>(Picture)</th>
<th>(Speech)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) VSB</td>
<td>VSB</td>
</tr>
<tr>
<td>(b) VSB</td>
<td>SSB</td>
</tr>
<tr>
<td>(c) VSB</td>
<td>FM</td>
</tr>
<tr>
<td>(d) FM</td>
<td>VSB</td>
</tr>
</tbody>
</table>

[GATE 1990: 2 Marks]

Soln. Note that VSB modulation is the clever compromise between SSB and DSB. Since TV bandwidth is large so VSB is used for picture transmission. Also, FM is the best option for speech because of better noise immunity.

Option (c)

2. In a double side-band (DSB) full carrier AM transmission system, if the modulation index is doubled, then the ratio of total sideband power to the carrier power increases by a factor of ________.

[GATE 2014: 1 Mark]

Soln. The AM system is Double side band (DSB) with full carrier. The expression for total power in such modulation signal is

\[ P_t = \frac{E_c^2}{2R} + \frac{\mu^2 E_c^2}{4} + \frac{\mu^2 E_c^2}{4} \]
or, \( P_t = P_c + \frac{\mu^2}{2} P_c \)

The second term on the right hand side is side band power.

so, \( P_{SB} = \frac{\mu^2}{2} P_c \)

or, \( \frac{P_{SB}}{P_c} = \frac{\mu^2}{2} \)

Now if \( \mu \) (modulation index) is doubled then \( \frac{P_{SB}}{P_c} \) will be 4 times

So, it is factor of 4

Ans. Factor of 4

3. The maximum power efficiency of an AM modulator is
   (a) 25%         (c) 33%
   (b) 50%         (d) 100%

   [GATE 1992: 2 Marks]

Soln. Efficiency of modulation can be given as

\[
\eta = \frac{P_s}{P_c + P_s} = \frac{\frac{\mu^2}{2} P_c}{P_c + \frac{\mu^2}{2} P_c}
\]

\[
\frac{\frac{\mu^2}{2}}{1 + \frac{\mu^2}{2}} = \frac{\mu^2}{(2 + \mu^2)}
\]
\[ \mu = 1 \text{ is the optimum value} \]

\[ s.o., \eta = \frac{1}{2 + 1} = \frac{1}{3} \times 100 = 33\% \]

Option (c)

4. Consider sinusoidal modulation in an AM systems. Assuming no over modulation, the modulation index (\( \mu \)) when the maximum and minimum values of the envelope, respectively, are 3V and 1V is \[ \text{[GATE 2014: 1 Mark]} \]

Soln. As given is the problem the modulation is sinusoidal this is also called tone modulation.

There is no over modulation means that modulation index is less than or equal to 1.

In such case the formula for modulation index is given by

\[ \mu = \frac{E_{\text{max}} - E_{\text{min}}}{E_{\text{max}} + E_{\text{min}}} \]

Where \( E_{\text{max}} \) is the maximum value of the envelope

\( E_{\text{min}} \) is the minimum value of the envelope.

This method is popular when the modulated waveform is observed is CRO

\[ \mu = \frac{3 - 1}{3 + 1} = \frac{2}{4} = \frac{1}{2} = 0.50 \]

Modulation index is 0.50
5. Which of the following analog modulation scheme requires the minimum transmitted power and minimum channel band-width?

(a) VSB  
(b) DSB-SC  
(c) SSB  
(d) AM  

[GATE: 2005 1 Mark]

Soln.  

<table>
<thead>
<tr>
<th>Modulation type</th>
<th>BW</th>
<th>Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional AM</td>
<td>$2f_m$</td>
<td>Maximum power</td>
</tr>
<tr>
<td>DSB SC</td>
<td>$2f_m$</td>
<td>(Less power)</td>
</tr>
<tr>
<td>VSB</td>
<td>$f_m +$ vestige</td>
<td></td>
</tr>
<tr>
<td>SSB</td>
<td>$f_m$</td>
<td>Less &amp; power</td>
</tr>
</tbody>
</table>

So, SSB least power & bandwidth  

Option (c)

6. Suppose that the modulating signal is $m(t) = 2 \cos(2\pi f_m t)$ and the carrier signal is $x_c(t) = A_c \cos(2\pi f_c t)$. Which one of the following is a conventional AM signal without over-modulation?

(a) $x(t) = A_c m(t) \cos(2\pi f_c t)$  
(b) $x(t) = A_c [1 + m(t)] \cos(2\pi f_c t)$  
(c) $x(t) = A_c \cos(2\pi f_c t) + \frac{A_c}{4} m(t) \cos(2\pi f_c t)$  
(d) $x(t) = A_c \cos(2\pi f_m t) \cos(2\pi f_c t) + A_c \sin(2\pi f_m t) \sin(2\pi f_c t)$  

[GATE 2010: 1 Mark]

Soln.  

Given  

Modulation signal $m(t) = 2 \cos(2\pi f_m t)$  
Carrier signal $x_c(t) = A_c \cos(2\pi f_c t)$  

Note that conventional AM is DSB – FC (DSB full carrier)  

Standard Expression is given by
\[
e(t) = E_c[1 + m(t)] \cos \omega_c t
\]

Or \[
e(t) = E_c[1 + \mu \cos \omega_m t] \cos \omega_c t
\]  \( \cdots \cdots \) (1)

Option (b) is \[
x(t) = A_c[1 + 2 \cos(2\pi f_m t)] \cos 2\pi f_c t
\]

Comparing this expression with the standard one given equation (I)

We get \( \mu = 2 \) i.e. conventional AM with over modulation

Option (c)

\[
x(t) = A_c \cos 2\pi f_c t + \frac{A_c}{4} m(t) \cos 2\pi f_c t
\]

\[
= A_c \left[ 1 + \frac{1}{4} \cdot 2 \cos(2\pi f_m t) \cos 2\pi f_c t \right]
\]

\[
= A_c \left[ 1 + \frac{1}{2} \cos(2\pi f_m t) \right] \cos 2\pi f_c t
\]

Here \( \mu = \frac{1}{2} \)

So, this represents conventional AM without over modulation.

Option (d) is non standard expression

So, correct option is option (c)

7. For a message signal \( m(t) = \cos(2\pi f_m t) \) and carrier of frequency \( f_c \). Which of the following represents a single side-band (SSB) signal?

(a) \( \cos(2\pi f_m t) \cos(2\pi f_c t) \)
(b) \( \cos(2\pi f_c t) \)
(c) \( \cos[2\pi (f_c + f_m) t] \)
(d) \( [1 + \cos(2\pi f_m t)]. \cos(2\pi f_c t) \)

[GATE 2009: 1 Mark]
Soln. Option (a) in the problem represents AM signal DSB-SC. If will have both side bands

option (b) represents only the carrier frequency

Option (c), \( \cos[2\pi(f_c + f_m)t] \) represents upper side band (SSB-SC). It represent SSB signal

Option (d) represents the conventional AM signal

Ans. Option (c)

8. A DSB-SC signal is generated using the carrier \( \cos(\omega_c t + \theta) \) and modulating signal \( x(t) \). The envelop of the DSB-SC signal is

(a) \( x(t) \)

(b) \(|x(t)|\)

(c) Only positive portion of \( x(t) \)

(d) \( x(t) \cos \theta \)

[GATE 1998: 1 Mark]

Soln. Given

Carrier \( c(t) = \cos(\omega_c t + \theta) \)

Modulating signal \( m(t) = x(t) \)

DSB SC modulated signal is given by \( c(t).m(t) = s(t) \)

\[ = x(t) \cos(\omega_c t + \theta) \]

\[ = x(t)\{\cos \theta . \cos \omega_c t - \sin \theta \sin \omega_c t\} \]

\[ = x(t) \cos \theta . \cos \omega_c t - x(t) . \sin \theta \sin \omega_c t \]

Envelope of \( s(t) = \sqrt{[x(t) \cos \theta]^2 + [x(t) \sin \theta]^2} \)

\[ = \sqrt{x^2(t)(\cos^2 \theta + \sin^2 \theta)} \]

\[ = x(t) \]

Option (b) \(|x(t)|\)
9. A 1 MHz sinusoidal carrier is amplitude modulated by a symmetrical square wave of period 100 µsec. Which of the following frequencies will not be present in the modulated signal?
   (a) 990 kHz  
   (b) 1010 kHz  
   (c) 1020 kHz  
   (d) 1030 kHz

   [GATE 2002: 1 Mark]

Soln. Frequency of carrier signal is $1 MHz = 1000 KHz$

Modulation signal is square wave of period 100 µS.

Frequency $= \frac{1}{100 \times 10^{-6}} = 10 KHz$

Since modulation signal is symmetrical square wave it will contain only odd harmonics i.e. 10 KHz, 30 KHz, 50 KHz -----etc.

Thus the modulated signal has

$$f_c \pm f_m = (1000 \pm 10 KHz) = 1010 KHz \& 990 KHz$$

$$f_c \pm 3f_m = (1000 \pm 30 KHz) = 1030 KHz \& 970 KHz$$

So, 1020 KHz will not be present in modulated signal

Option (c)

10. A message signal given by $m(t) = \left(\frac{1}{2}\right) \cos \omega_1 t - \left(\frac{1}{2}\right) \sin \omega_2 t$ is amplitude modulated with a carrier of frequency $\omega_c$ to generate $s(t) = [1 + m(t)] \cos \omega_c t$

What is the power efficiency achieved by this modulation scheme?
   (a) 8.33%  
   (b) 11.11%  
   (c) 20%  
   (d) 25%

   [GATE 2009: 2 Marks]
Soln. Given

\[ m(t) = \frac{1}{2} \cos \omega_1 t - \frac{1}{2} \sin \omega_2 t \]

\[ s(t) = [1 + m(t)] \cos \omega_c t \]

Note that the modulation frequency are \( \omega_1 \) and \( \omega_2 \) i.e. multitone modulation

Net modulation index is \( \mu = \sqrt{\mu_1^2 + \mu_2^2 + \cdots + \mu_n^2} \)

Here, \( \mu = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{1}{\sqrt{2}} \)

\[ \eta = \frac{\mu^2}{\mu^2 + 2} \times 100\% \]

\[ = \frac{(1/\sqrt{2})^2}{(1/\sqrt{2})^2 + 1} \times 100\% = 20\% \]

Option (c)

11. A 4 GHz carrier is DSB-SC modulated by a low-pass message signal with maximum frequency of 2 MHz. The resultant signal is to be ideally sampled. The minimum frequency of the sampling impulse train should be

(a) 4 MHz
(b) 8 MHz
(c) 8 GHz
(d) 8.004 GHz

[GATE: 1990 2 Mark]

Soln. Given

\[ f_c = 4 \text{ GHz} = 4000 \text{ MHz} \]

\[ f_m = 2 \text{ MHz (low pass message signal)} \]

Such a signal is amplitude modulated (DSB-SC) i.e. two side bands \( (f_c + f_m) \& (f_c - f_m) \)
i.e. 4002 & 3998 or 4 MHz = BW

so, min. sampling frequency should be (Nyquist Rate)

option (b) \( f_{s(min)} = 2 \times 4 = 8 \text{ MHz} \)

(b) **Demodulation**

12. Consider the amplitude modulated (AM) signal \( A_c \cos \omega_c t + 2 \cos \omega_m t \cos \omega_c t \). For demodulating the signal using envelope detector, the minimum value of \( A_c \) should be

(a) 2 \quad (b) 1 \quad (c) 0.5 \quad (d) 0

[\text{GATE 2008: 1 Mark}]

Soln. Modulated signal is given as

\[ \varphi_{AM}(t) = A_c \cos \omega_c t + 2 \cos \omega_m t \cos \omega_c t \]

\[ \varphi_{AM}(t) = [A_c + 2 \cos \omega_c t] \cos \omega_m t \]

Note that for envelope detection the modulation should not go beyond full modulation i.e. \( \mu = 1 \), so amplitude of baseband signal has to be less than the carrier amplitude \( A_c \)

\[ |f(t)|_{max} \leq A_c \]

i.e. \( |2 \cos \omega_m t|_{max} = 2 \leq A_c \)

or \( A_c \geq 2 \)

option (a)

13. Which of the following demodulator(s) can be used for demodulating the signal
\[ x(t) = 5(1 + 2 \cos 200 \pi t) \cos 2000\pi t \]

(a) Envelope demodulator  
(b) Square-law demodulator  
(c) Synchronous demodulator  
(d) None of the above  

\[ \text{Soln. The modulated signal given is} \ x(t) = 5(1 + 2 \cos 200 \pi t) \cos 2000\pi t \]

The standard equation for AM is

\[ X_{\text{AM}}(t) = A_c (1 + \mu \cos \omega_m t) \cos \omega_c t \]

If we compare the two equations we find \( \mu = 2 \).

The modulation index is more than 1 here, so it is the case of over modulation.

When modulation index is more than 1 (over modulation) then detection is possible only with, Synchronous modulation, such signal can not be detected with envelope detector.

Option (c)

14. The amplitude modulated wave form \( s(t) = A_c [1 + K_a m(t)] \cos \omega_c t \) is fed to an ideal envelope detector. The maximum magnitude of \( K_a m(t) \) is greater than 1. Which of the following could be the detector output ?

(a) \( A_c m(t) \)  
(b) \( A_c^2 [1 + K_a m(t)]^2 \)  
(c) \( |A_c [1 + K_a m(t)]| \)  
(d) \( A_c |1 + K_a m(t)|^2 \)  

\[ \text{Soln. Given} \]

\[ |K_a m(t)| > | \]

For the above condition the AM signal is over modulated. Envelope detector will not be able to detect over modulated signal correctly.

Non of the above options
15. The diagonal clipping in Amplitude Demodulation (using envelope detector) can be avoided if RC time-constant of the envelope detector satisfies the following condition, (here W is message bandwidth and ω is carrier frequency both in rad/sec)

(a) \( RC < \frac{1}{W} \)  
(b) \( RC > \frac{1}{W} \)

(c) \( RC < \frac{1}{\omega} \)  
(d) \( RC > \frac{1}{\omega} \)

[GATE 2006: 2 Marks]

Soln. It is seen that to avoid negative peak clipping also said diagonal clipping the RC time constant of detector should be

Or \( \tau < \frac{1}{f_m} \)

Note \( f_m \) is maximum modulating frequency i.e. the bandwidth \( w \)

So, \( RC < \frac{1}{w} \)

16. An AM signal is detected using an envelope detector. The carrier frequency and modulation signal frequency are 1 MHz and 2 KHz respectively. An appropriate value for the time constant of the envelope detector is

(a) 500 µsec  
(b) 20 µsec

(c) 0.2 µsec  
(d) 1 µsec

[GATE 2004: 1 Mark]

Soln. Note that the time constant RC should satisfy the following condition

\[ \frac{1}{f_c} < RC < \frac{1}{f_m} \]

\[ \frac{1}{1\times10^6} < RC < \frac{1}{2\times10^3} \]

Or \( 1 \mu s < RC < 0.5ms \)

Option (b)
17. A DSB-SC signal is to be generated with a carrier frequency \( f_c = 1 MHz \) using a non-linear device with the input-output characteristic

\[
V_0 = a_0 v_i + a_1 v_i^3
\]

Where \( a_0 \) and \( a_1 \) are constants. The output of the non-linear device can be filtered by an appropriate band-pass filter.

Let \( V_i = A_c^i \cos(2\pi f_c^i t) + m(t) \) where \( m(t) \) is the message signal. Then the value of \( f_c^i \) (in MHz) is

(a) 1.0 
(b) 0.333 
(c) 0.5 
(d) 3.0

[GATE 2003: 2 Marks]

Soln. \[ V_0 = \varphi_0[A_c^i \cos(2\pi f_c^i t) + m(t)] \]

\[ + \varphi_1[A_c \varphi_{AM}(t) = A_c \cos \omega_c t \cdot 2 \cos \omega_c t] \]

\[ = \varphi_0[A_c^i \cos(2\pi f_c^i t) + m(t)] \]

\[ + \varphi_1 \left[ (A_c^i)^3 \cos^3(2\pi f_c^i t) + m^3(t) \right] \]

\[ + 3. A_c^i \cos(2\pi f_c^i t) \cdot m^2(t) + 3. (A_c^i)^2 \cos^2(2\pi f_c^i t) \cdot m(t) \]

AM – DSB – SC signal lies is

\[ \varphi_1. 3(A_c^i)^2 m(t) \cos^2(\pi f_c^i t) \]

For DSB – SC the last term is important

\[ 3 \varphi_1 (A_c^i)^2 \cos^2 2\pi f_c^i t \cdot m(t) \]

\[ 3 \varphi_1 (A_c^i)^2 \cdot m(t) \cdot [1 + \cos 2\pi (2f_c^i t)] \]

Note \( m(t) \cos \omega_c t \rightarrow f_c \) (carrier) = 1MHz

For \( \cos^2 \) term as expended the term is having \( 2f_c^i \)

\[
2f_c^i = 1 MHz \quad \text{so, } \quad f_c^i = 0.5 MHz
\]

Option (c)
18. A message signal \( m(t) = \cos 2000 \pi t + 4 \cos 4000 \pi t \) modulates the carriers \( c(t) = \cos 2\pi f_c t \) where \( f_c = 1 MHz \) to produce an AM signal. For demodulating the generated AM signal using an envelope detector, the time constant \( RC \) of the detector circuit should satisfy

(a) \( 0.5 \text{ ms} < RC < 1 \text{ ms} \)  
(b) \( 1 \mu s << RC < 0.5 \text{ ms} \)  
(c) \( RC << 1 \mu s \)  
(d) \( RC >> 0.5 \text{ ms} \)

[GATE 2011: 2 Marks]

**Soln.** Message signal is

\[ m(t) = \cos 2000\pi t + 4 \cos 4000\pi t \]

It consist of two frequencies \( \omega_1 = 2000\pi \)

Or \( 2\pi f_1 = 2000\pi \)

Or \( f_1 = 1 KHz \)

\[ f_2 = 2 KHz \]

So, Max frequency is 2 KHz

So, \( \frac{1}{f_c} << RC < \frac{1}{f_m} \)

\[ \frac{1}{1 MHz} << RC < \frac{1}{2 KHz} \]

Or, \( 1 \mu s << RC < 0.5 \text{ ms} \)

Option (b)
19. A super heterodyne radio receiver with an intermediate frequency of 455 KHz is tuned to a station operating at 1200 KHz. The associated image frequency is ..........KHz

[GATE 1993: 2 Marks]

Soln. In most receivers the local oscillator frequency is higher than incoming signal i.e.

\[ f_0 (\text{frequency of local oscillator}) = f_s + f_i \]

Where \( f_s \)-------- signal frequency

\( f_i \)or \( f_{si} \)-------- Image frequency

\[ f_{si} = f_s + 2IF = f_s + 2f_i \]

\[ f_{si} = 1200 + 2(455) \]

\[ f_{si} = 2110 \text{ KHz} \]

so, answer is 2110 KHz
20. The image channel selectivity of superheterodyne receiver depends upon
(a) IF amplifiers only
(b) RF and IF amplifiers only
(c) Pre selector, RF and IF amplifiers
(d) Pre selector and RF amplifiers

[GATE 1998: 1 Marks]

Soln. Image rejection depends on front end selectivity of receiver and must be
achieved before IF stage. So image channel selectivity depends upon pre
selector & RF amplifier. If it enters IF stage it becomes impossible to
remove it from wanted signal.

Option (d)

Only problem statement is different otherwise they are same problems.