Continuous Time Signals (Part - I) – Fourier series

(a) Basics

1. Which of the following signals is/are periodic?
   (a) $s(t) = \cos 2t + \cos 3t + \cos 5t$
   (b) $s(t) = \exp(j8\,\pi t)$
   (c) $s(t) = \exp(-7t)\sin 10\pi t$
   (d) $s(t) = \cos 2t \cos 4t$

[GATE 1992: 2 Marks]

Soln. (a) $s(t) = \cos 2\,t + \cos 3\,t + \cos 5\,t$

First term has $\omega_1 = 2$

Second term has $\omega_2 = 3$

Third term has $\omega_3 = 5$

Note that ratio of any two frequencies equals p/q is rational where p and q are integers.

Thus $s(t)$ is periodic

(b) $s(t) = \exp(j8\pi t)$

$= \cos(8\pi t) + j\sin(8\pi t)$

$$\frac{\omega_1}{\omega_2} = \frac{8\pi}{8\pi} = 1 \qquad so\ periodic$$

(c) $s(t) = \exp(-7t)\,.\sin 10\pi t$

$= e^{-7t}.\sin 10\pi t$

$$Note, \quad T = \frac{2\pi}{\omega} = \frac{2\pi}{10\pi} = \frac{1}{5}$$

Due to e^{-7t} it is decaying function, so not periodic

(d) $s(t) = \cos 2\,t\,.\cos 4\,t$

Note, $2\cos A \cos B = \cos(A - B) + \cos(A + B)$

$so,\ s(t) = \frac{1}{2}[\cos 2\,t + \cos 6\,t]$

$$Note, \quad \frac{p}{q} = \frac{\omega_1}{\omega_2} = \frac{2}{6} = \frac{1}{3}$$

Rational

So, (a) , (b) and (d) are periodic.

2. The power in the signal
$s(t) = 8cos\left(20\pi t - \frac{\pi}{2}\right) + 4sin(15\pi t)\ is$

(a) 40 (c) 42
(b) 41 (d) 82

[GATE 2005: 1 Mark]

Soln. Time average of energy of a signal = Power of Signal

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} f^2(t)dt = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |f(t)|^2 dt$$

Signal power P is mean of the signal amplitude squared value of f(t) . Rms value of signal $= \sqrt{P}$

$$S(t) = 8\cos\left(20\pi t - \frac{\pi}{2}\right) + 4\sin(15\pi t)$$

$$= 8\sin(20\pi t) + 4\sin(15\pi t)$$

$$= \frac{8^2}{2} + \frac{4^2}{2} = 32 + 8 = 40$$

Option (a)

3. If a signal f(t) has energy E, the energy of the signal f(2t) is equal to

(a) E
(b) E/2
(c) 2E
(d) 4E

[GATE 2001: 1 Mark]

Soln. Energy of a signal is given by

$$E = \int_{-\infty}^{\infty} [f(t)]^2 dt$$

Energy of the signal f(2t) is

$$E_s = \int_{-\infty}^{\infty} [f(t)]^2 \, dt$$

Let $2t = p \;\; or, \;\; dt = \frac{dp}{2}$

$$= \int_{-\infty}^{\infty} [f(t)]^2 \, \frac{dp}{2}$$

$$E_s = \frac{E}{2}$$

Option (b)

4. For a periodic signal
$v(t) = 30 \sin 100t + 10 \cos 300t + 6 \sin(500t + \frac{\pi}{4})$, the fundamental frequency in rad/s is

(a) 100
(b) 300
(c) 500
(d) 1500

[GATE 2013: 1 Mark]

Soln. First term has $\omega_1 = 100$

Second term $\omega_2 = 300$

Third term $\omega_3 = 500$

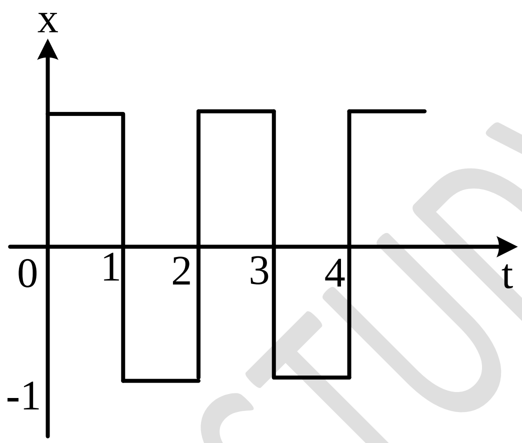
ω_1 is the fundamental frequency

ω_2 is third harmonic

ω_3 is 5^{th} harmonic

Option (a)

5. Consider the periodic square wave in the figure shown



The ratio of the power in the 7^{th} harmonic to the power in the 5^{th} harmonic for this waveform is closest in value to ------

[GATE 2014: 1 Mark]

Soln. For a periodic square wave n^{th} harmonic component $\propto \frac{1}{n}$

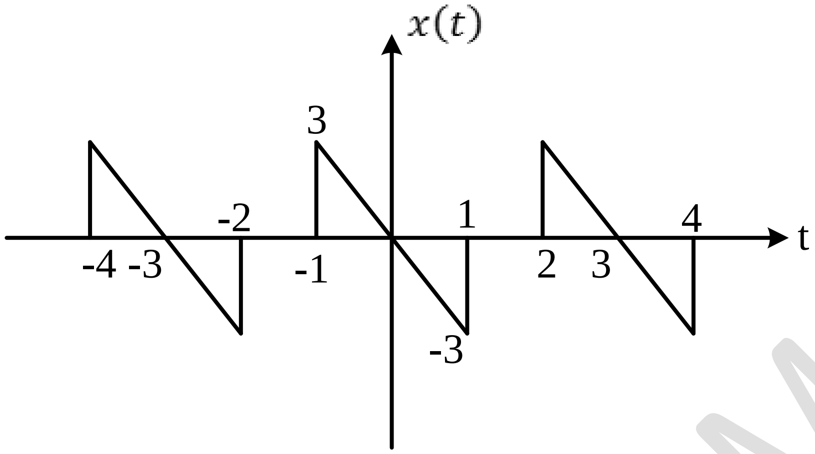
Thus the power in the n^{th} harmonic component is $\propto {}^{1}/_{n^2}$

Ratio of power in 7^{th} harmonic to 5^{th} harmonic for the given wage form is

$$\frac{{}^{1}/_{7^2}}{{}^{1}/_{5^2}} = \frac{25}{29} \cong 0.5$$

Answer 0.5

6. The waveform of a periodic signal $x(t)$ is shown in the figure.



A signal $g(t)$ is defined by $g(t) = x\left(\frac{t-1}{2}\right)$. The average power of $g(t)$ is _________ .

[GATE 2015: 1 Mark]

Soln. The equation for the given waveform can be written as

$$x = -3\,t$$

The period of the waveform is 3 (i.e. from -1 to +2)

$$Av.Power = \frac{1}{T}\int [x(t)]^2\,dt$$

$$= \frac{1}{3}\left[\int_{-1}^{0} (-3\,t)^2\,dt + \int_{0}^{1} (-3\,t)^2\,dt + \int_{1}^{2} 0^2.dt\right]$$

$$= \frac{1}{3}\left[9.\frac{t^3}{3}\Big|_{-1}^{0} + 9\,\frac{t^3}{3}\Big|_{0}^{1} + 0\right]$$

$$= \frac{1}{3}\left[\frac{9}{3}.\{0-(-1)\} + \frac{9}{3}(1-0)\right]$$

$$= \frac{1}{3}\left[\frac{9}{3} + \frac{9}{3}\right] = \frac{1}{3} \times \frac{18}{3} = 2$$

Answer 2

7. The RMS value of a rectangular wave of period T, having a value of +V for a duration $T_1\ (< T)\ and\ -V$ for the duration $T - T_1 = T_2$, equals

(a) V

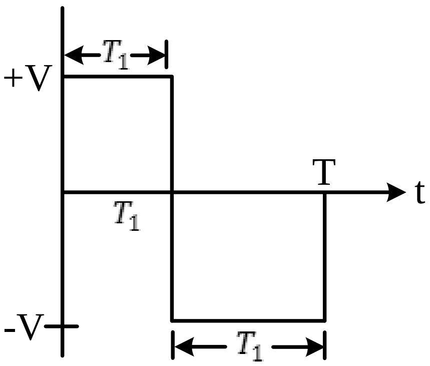
(b) $\frac{T_1-T_2}{T}V$

(c) $\frac{V}{\sqrt{2}}$

(d) $\frac{T_1}{T_2}V$

[GATE: 1995 1 Mark]

Soln.



The waveform can be drawn as per the given problem.

Period $(T) = T_1 + T_2$

$$RMS\ value = \sqrt{\frac{1}{T}\int_0^T x^2(t)\,dt}$$

$$= \sqrt{\frac{1}{T}\left[\int_0^{T_1} V^2\,dt + \int_{T_1}^{T} (-V)^2\,dt\right]}$$

$$= \sqrt{\frac{1}{T}[V^2.(T_1 - 0) + V^2(T - T_1)]}$$

$$= \sqrt{\frac{1}{T}.V^2\,[\,T_1 + T - T_1]}\ =\ \sqrt{V^2} = V \qquad Option\ (a)$$

(b) Fourier series

8. The trigonometric Fourier series of an even function of time does not have
   (a) the dc term
   (b) cosine terms
   (c) sine terms
   (d) odd harmonic terms

[GATE 1996: 1 Mark]

Soln. For periodic even function, the trigonometric Fourier series does not contain the sine terms (odd functions)

It has dc term and cosine terms of all harmonics.

Option (c)

9. The trigonometric Fourier series of a periodic time function can have only
   (a) cosine terms
   (b) sine terms
   (c) cosine and sine terms
   (d) dc and cosine terms

[GATE 1998: 1 Mark]

Soln. The Fourier series of a periodic function $x(t)$ is given by the form

$$x(t) = \sum_{n-0}^{\infty} a_n \cos n\, \omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\, \omega_0\, t$$

Thus the series has cosine terms of all harmonics: $n\omega_0, n = 0,1,2 - - -$

Where 0^{th} harmonic = dc term (average or mean) = a_0 and sine terms of all harmonics: $n\omega_0, n = 1,2, - - - -$.

10. The Fourier series of an odd periodic function, contains only
   (a) odd harmonics
   (b) even harmonics
   (c) cosine terms
   (d) sine terms

[GATE 1994: 1 Mark]

Soln. If periodic function is odd the dc term $a_0 = 0$ and also cosine terms (even symmetry)

It contains only sine terms

Option (d)

11. The Fourier series of a real periodic function has only

P. Cosine terms if it is even
Q. Sine terms if it is even
R. Cosine terms if it is odd
S. Sine terms if it odd

Which of the above statements are correct?

(a) P and S
(b) P and R
(c) Q and S
(d) Q and R

[GATE 2009: 1 Mark]

Soln. The Fourier series for a real periodic function has only cosine terms if it is even and sine terms if it is odd

Option (a)

12. The trigonometric Fourier series of an even function does not have the

(a) dc term
(b) cosine terms
(c) sine terms
(d) odd harmonic terms

[GATE 2011: 1 Mark]

Soln. The trigonometric Fourier series of an even function has cosine terms which are even functions.

It has dc term if its average value is finite and no dc term if average value is zero

So it does not have sine terms

Option (c)

13. Which of the following cannot be the Fourier series expansion of a periodic signals?

(a) $x(t) = 2\cos t + 3\cos 3t$
(b) $x(t) = 2\cos \pi t + 7\cos t$
(c) $x(t) = \cos t + 0.5$
(d) $x(t) 2\cos 1.5\pi t + \sin 3.5\pi t$

[GATE 2002: 1 Mark]

Soln. (a) $x(t) = 2\cos t + 3\cos t$ is periodic signal with fundamental frequency $\omega_0 = 1$

(b) $x(t) = 2\cos \pi t + 7\cos t$ The frequency of first term $\omega_1 = \pi$ frequency of 2nd term is $\omega_2 = 1$

$$\frac{\omega_1}{\omega_2} = \frac{\pi}{1} \; is\ not\ the\ rational\ number$$

So $x(t)$ is aperiodic or not periodic

(c) $x(t) = \cos t + 0.5$ is a periodic function with $\omega_0 = 1$

(d) $x(t) = 2\cos(1.5\pi)t + \sin(3.5\pi)t$ first term has frequency $\omega_1 = 1.5\pi$ 2nd term has frequency $\omega_2 = 3.5\pi$

$$\frac{\omega_1}{\omega_2} = \frac{1.5\pi}{3.5\pi} = \frac{1.5}{3.5} = \frac{3 \times 0.5}{7 \times 0.5} = \frac{3}{7}$$

So about ratio is rational number $x(t)$ is a periodic signal, with fundamental frequency $\omega_0 = 0.5\pi$

Since function in (b) is non periodic. So does not satisfy Dirictilet condition and cannot be expanded in Fourier series

14. Choose the function $f(t), -\infty < t < \infty$, for which a Fourier series cannot be defined.
(a) $3\sin(25t)$
(b) $4\cos(20t + 3) + 2\sin(710t)$
(c) $\exp(-|t|)\sin(25t)$
(d) 1

[GATE 2005: 1 Mark]

Soln. Fourier series is defined for periodic function and constant

(a) $3\sin(25\,t)\ is\ periodic\ \omega = 25$
(b) $4\cos(20\,t + 3) + 2\sin(710\,t)$ sum of two periodic function is also periodic function
(c) $e^{-|t|}\sin 25\,t$ Due to decaying exponential decaying function it is not periodic. So Fourier series cannot be defined for it.
(d) Constant, Fourier series exists.
Fourier series can't be defined for option (c)

15. A periodic signal $x(t) of\ period\ T_0$ is given by

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, T_1 < |t| < \dfrac{T_0}{2} \end{cases}$$

The dc component of $x(t)$ is

(a) $\frac{T_1}{T_0}$

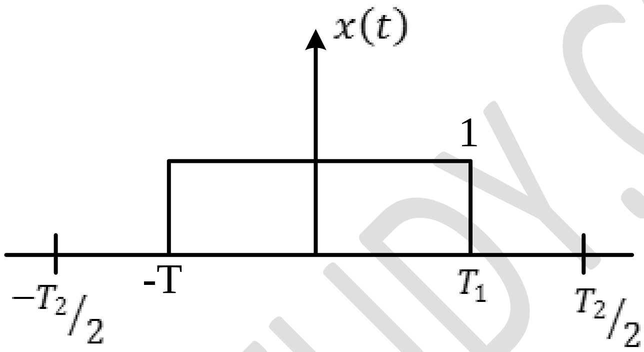
(b) $\frac{T_1}{(2T_0)}$

(c) $\frac{2T_1}{T_0}$

(d) $\frac{T_0}{T_1}$

[GATE 1998: 1 Mark]

Soln.



Given periodic signal can be drawn having period T_0

Fourier series the function $x(t)$ can be written as

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_o t + b_n \sin n\omega_0 t$$

Where dc component given by

$$a_0 = \frac{1}{T_0} \int_{T_0}^{\infty} x(t)\, dt$$

$$a_0 = \frac{1}{T} \int_{-T_0/2}^{T_0/2} x(t) dt$$

$$= \frac{1}{T_0}\left[\int_{-T_0/2}^{-T_1} x(t)dt + \int_{-T_1}^{T_1} x(t)dt + \int_{T_1}^{T_0/2} x(t)dt\right]$$

$$= \frac{1}{T_0}[0 + 2T_1 + 0]$$

$$= \frac{2T_1}{T_0}$$

Option (c)

16. The Fourier series representation of an impulse train denoted by

$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0) \ \ is\ given\ by$$

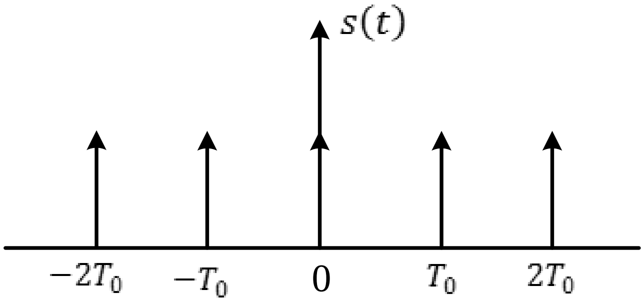
(a) $\left(\frac{1}{T_0}\right)\sum_{n=-\infty}^{\infty} \exp(-j2\pi n\, t/T_0)$

(b) $\left(\frac{1}{T_0}\right)\sum_{n=-\infty}^{\infty} \exp(-j\pi n\, t/T_0)$

(c) $\left(\frac{1}{T_0}\right)\sum_{n=-\infty}^{\infty} \exp(j\pi n\, t/T_0)$

(d) $\left(\frac{1}{T_0}\right)\sum_{n=-\infty}^{\infty} \exp(j2\pi n\, t/T_0)$

Soln.



The given impulse train $s(t)$ with strength of each impulse as 1 is aperiodic function with period T_0

$$s(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} \quad where\ \omega_0 = \frac{2T}{T_0}$$

$$where\ C_n = \frac{1}{T_0} \int_{-T_0/2}^{+T_0/2} \delta(t) e^{-jn\omega_0 t}\, dt = \frac{1.\, e^{-jn\omega_0 t}}{T_0}\Big| t = 0 \quad = {}^{1}/_{T_0}$$

17. The Fourier series expansion of a real periodic signal with fundamental frequency f_0 is given by

$$g_p(t) = \sum_{n=-\infty}^{\infty} C_n\, e^{j\, 2\pi n\, f_0\, t}$$

It is given that $C_3 = 3 + j5 \quad then \quad C_{-3} \quad is$

(a) $5 + 3j$
(b) $-3 - j5$
(c) $-5 + 3j$
(d) $3 - j5$

[GATE 2003: 1 Mark]

Soln. Given $C_3 = 3 + j5$

We know that for real periodic signal

$$C_{-k} = C_k^*$$

$$So, \quad C_{-3} = C_3^* = (3 - j5)$$