Continuous Time Signals (Part - II) - Fourier Transform

1. The Fourier transform of a real valued time signal has
   (a) odd symmetry   (c) conjugate symmetry
   (b) even symmetry  (d) no symmetry

   [GATE 1996: 1 Mark]

Soln. For real valued time signal, Fourier Transform has conjugate symmetry.

If \( x(t) \) is real \( \rightarrow \) Fourier Transform is \( X(f) \)

Then there exists conjugate even symmetry (Also called Hermitian Symmetry)

i.e. \( X(f) = X^*(-f) \)

or \( X^*(f) = X(-f) \)

From above condition it can be shown that

\( |X(f)| \) and \( Re\{X(f)\} \) have even symmetry

i.e. \( |X(f)| = |X(-f)| \)

\( \angle X(f)\) and \( Im\{X(f)\} \) have odd symmetry

\( \angle X(f) = -\angle X(-f) \)

Option (c)

2. A signal \( x(t) \) has a Fourier transform \( X(\omega) \). If \( x(t) \) is a real and odd function of \( t \), then \( X(\omega) \) is
   (a) a real and even function of \( \omega \)
   (b) an imaginary and odd function of \( \omega \)
   (c) an imaginary and even function of \( \omega \)
   (d) a real and odd function of \( \omega \)

   [GATE 1999: 1 Mark]

Soln. If \( f(t) \) is real and even then \( F(\omega) \) is real

Even \( \rightarrow f(t) = f(-t) \)

\( F(\omega) = F(-\omega) \)

Real \( \rightarrow f(-\omega) = f^*(\omega) \)

Or \( F(\omega) = F^*(\omega) \)
If $f(t)$ is real and odd

$F(\omega)$ is pure imaginary

odd $\rightarrow f(t) = -f(-t)$

$F(\omega) = -F(-\omega)$

Option (b)

3. The Fourier transform of a conjugate symmetric function is always

(a) imaginary  
(b) conjugate anti-symmetric  
(c) real  
(d) conjugate symmetric  

[GATE 2004: 1 Mark]

Soln. Given that the time function $x(t)$ is conjugate symmetric i.e.

If $x(t) = x^*(-t)$

Use the property of conjugate symmetry of FT

If $x(t) \rightarrow X(f)$

Then $x^*(-t) = X^*(f)$

Given $x(t) = x^*(-t)$

Then $X(f) = X^*(f)$

So, $X(f)$ is real

Option (c)

4. If $G(f)$ represents the Fourier Transform of a signal $g(t)$ which is real and odd symmetric in time, then

(a) $G(f)$ is complex  
(b) $G(f)$ is imaginary  
(c) $G(f)$ is real  
(d) $G(f)$ is real  

[GATE 1992: 2 Marks]

Soln. $g(t) \rightarrow G(f)$

Note, If $g(t)$ is real and even, $G(f)$ is also real and even

But if $g(t)$ is real and odd
\( G(f) \) is imaginary and odd

Option (b)

5. The amplitude spectrum of a Gaussian pulse is
   (a) uniform  (c) Gaussian
   (b) a sine function  (d) An impulse function

   [GATE 1998: 1 Mark]

Soln. Gaussian pulse is defined by
   \[ f(t) = e^{-\pi t^2} \]

Fourier Transform of this pulse can be evaluated

\[ \mathcal{F}[e^{-\pi t^2}] = \int_{-\infty}^{\infty} e^{-\pi t^2} e^{-j\omega t} dt \]

After evaluation of integral one gets

\[ \mathcal{F}[e^{-\pi t^2}] = e^{-\pi f^2} \]

When area under Gaussian pulse and central ordinate of the pulse is unity, it is said to be normalized Gaussian pulse. Such pulse is its own Fourier Transform

\[ e^{-\pi t^2} \leftrightarrow e^{-\pi f^2} \]

Option (c)

6. The Fourier Transform of the signal \( x(t) = e^{-3t^2} \) is of the following from where A and B are constants:
   (a) \( A e^{-B|f|} \)
   (b) \( A e^{-Bf} \)
   (c) \( A + B|f|^2 \)
   (d) \( A e^{-Bf^2} \)

   [GATE 2000: 1 Mark]

Soln. The Fourier Transform of a normalized Gaussian pulse is also normalized Gaussian pulse

For \( g(t) = e^{-at^2} \)
\[ G(\omega) = \sqrt{\frac{\pi}{a}} e^{-\omega^2/4a} \]

So, it is of the form 
\[ Ae^{-Bf^2} \]

The constants A and B can be found 

For \( x(t) = e^{-3t^2} \)

Here \( a = 3 \)

So, \( X(\omega) = \sqrt{\frac{\pi}{3}} . e^{-\omega^2/4 \times 3} \)

\[ X(\omega) = \sqrt{\frac{\pi}{3}} . e^{-\omega^2/12} \]

Option (d)

7. The function \( f(t) \) has Fourier Transform \( g(\omega) \). The Fourier Transform of

\[ g(t) = \left( \int_{-\infty}^{\infty} g(t) e^{-j\omega t} dt \right) \text{ is} \]

(a) \( \frac{1}{2\pi} f(\omega) \)  
(b) \( \frac{1}{2\pi} f(-\omega) \)  
(c) \( 2\pi f(\omega) \)  
(d) none of above  

[GATE 1997: 1 Mark]

Soln. Given 

\[ f(t) \overset{F}{\rightarrow} g(\omega) \]

Then \( F[g(t)] \) ?

Inverse transform

\[ f(t) = \frac{1}{2} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} \, d\omega \]
Or, \(2\pi f(-t) = \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega\)

Here \(\omega\) is dummy variable so can be exchanged

\[i.e \ 2\pi f(-\omega) = \int_{-\infty}^{\infty} F(t) e^{-j\omega t} dt = \mathcal{F}[f(t)]\]

Above equation shows that Fourier transform of time function \(f(t)\) is \(2\pi f(-\omega)\)

In this problem also, \(g(\omega)\) is Fourier Transform for \(f(t)\)

So changing dummy variable (from \(t\) to \(\omega\)) then \(\mathcal{F}\{g(t)\} = 2\pi f(-\omega)\)

Option (c)

8. The Fourier transform of a function \(x(t)\) is \(X(f)\). The Fourier transform of \(\frac{dx(t)}{dt}\) will be

(a) \(\frac{dX(f)}{dt}\)

(b) \(j 2\pi f X(f)\)

(c) \(j f X(f)\)

(d) \(\frac{X(f)}{jf}\)

[GATE 1998: 1 Mark]

Soln.

If \(x(t) \leftrightarrow X(f)\)

\[\text{then} \quad \frac{dx}{dt} \leftrightarrow j\omega X(\omega)\]

Since, \(x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)e^{j\omega t} d\omega\)

Then

\[\frac{dx(t)}{dt} = \frac{1}{2\pi} \cdot \frac{d}{dt} \left[ \int_{-\infty}^{\infty} X(\omega)e^{j\omega t} d\omega \right]\]
\[
\frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{d}{dt} \{X(\omega)e^{j\omega t}\} d\omega
\]

\[
= \frac{1}{2\pi} \int_{-\infty}^{\infty} j\omega X(\omega) e^{j\omega t} d\omega
\]

\[
= \mathcal{F}^{-1}[j\omega X(\omega)]
\]

\[
= \mathcal{F}^{-1}[j2\pi X(f)]
\]

This shows that differentiation in time domain is equivalent to multiplication by \(j\omega = j2\pi f\) in frequency domain.

Option (b)

9. The Fourier transform of a voltage signal \(x(t)\) is \(X(f)\). The unit of \(|X(f)|\) is
   (a) Volt
   (b) Volt – sec
   (c) Volt / sec
   (d) Volt²

   [GATE 1998: 1 Mark]

Soln. As per the definition of Fourier Transform

\[
X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt
\]

Looking at R.H.S expression, then unit of \(X(f)\) will be volt – sec

Option (b)

10. If a signal \(f(t)\) has energy \(E\), the energy of the signal \(f(2t)\) is equal to
   (a) \(E\)
   (b) \(E/2\)
   (c) \(2E\)
   (d) \(4E\)

   [GATE 2001: 1 Mark]

Soln. Given,

Signal \(f(t)\) has energy \(E\).
Find energy of the signal $f(2t)$.

Energy of signal

$$f(t) = \int_{-\infty}^{\infty} f^2(t)\,dt$$

So, energy of signal $f(2t)$ will be

$$= \int_{-\infty}^{\infty} f^2(2t)\,dt$$

$$= \int_{-\infty}^{\infty} f^2(\tau)\,\frac{d\tau}{2} = \frac{E}{2}$$

Option (b)

11. The Fourier transform $F\{e^{-t} u(t)\}$ is equal to

$$\frac{1}{1 + j2\pi f}$$

Therefore, $F\left\{\frac{1}{1+j2\pi t}\right\}$ is

- (a) $e^{f} u(f)$
- (b) $e^{-f} u(f)$
- (c) $e^{f} u(-f)$
- (d) $e^{-f} u(-f)$

[GATE 2002: 1 Mark]

Soln. Given,

$$\mathcal{F}[e^{-t} u(t)] = \frac{1}{(1+j2\pi f)}$$

Using the duality property

If $g(t) \rightarrow G(f)$

$$G(t) \rightarrow g(-f)$$

Therefore,
\[
\frac{1}{(1 + j 2\pi t)} \rightarrow e^f u(-f)
\]

Option (c)

12. Let \( x(t) \leftrightarrow X(j\omega) \) be Fourier Transform pair. The Fourier Transform of the signal \( x(5t - 3) \) in terms of \( X(j\omega) \) is given as

(a) \( \frac{1}{5} e^{-\frac{j3\omega}{5}} X\left(\frac{j\omega}{5}\right) \)

(b) \( \frac{1}{5} e^{\frac{j3\omega}{5}} X\left(\frac{j\omega}{5}\right) \)

(c) \( \frac{1}{5} e^{-j3\omega} X\left(\frac{j\omega}{5}\right) \)

(d) \( \frac{1}{5} e^{j3\omega} X\left(\frac{j\omega}{5}\right) \)

[GATE 2006: 1 Mark]

Soln. Given,

\( x(t) \leftrightarrow X(j\omega) \)

Find Fourier Transform of \( x(5t - 3) \)

Time shifting property

\( x(t + t_0) \rightarrow e^{\pm j\omega t_0} X(j\omega) \)

Scaling property

\( x(Kt) \rightarrow \frac{1}{|K|} X\left(\frac{j\omega}{K}\right) \)

Using time shifting property

\( x(t - 3) \rightarrow e^{-j3\omega} X(j\omega) \)

Using scaling property

\( x(5t - 3) = \frac{1}{5} e^{-\frac{j3\omega}{5}} \times \left(\frac{j\omega}{5}\right) \)

Option (a)
13. If the Fourier Transform of a deterministic signal \( g(t) \) is \( G(f) \), then

Items – 1
(1) The Fourier Transform of \( g(t - 2) \) is
(2) The Fourier Transform of \( g(t/2) \) is

Items – 2
(A) \( G(f)e^{-j(4\pi f)} \)
(B) \( G(2f) \)
(C) \( 2G(2f) \)
(D) \( G(f - 2) \)

Match each of the items 1, 2 on the left with the most appropriate item A, B, C, or D on the right.

[GATE 1997: 2 Marks]

Soln. 
\[ g(t) \leftrightarrow G(f) \]
\[ g(t - 2) \leftrightarrow e^{-j2\pi f} G(f) = G(f)e^{-j(4\pi f)} \]
\[ g\left(\frac{t}{2}\right) \leftrightarrow \left(\frac{1}{1/2}\right) G\left(\frac{f}{1/2}\right) = 2G(2f) \]

Option 1 – A, 2 – C

14. Let \( x(t) \) and \( y(t) \) (with Fourier transform \( X(f) \) and \( Y(f) \) respectively) be related as shown in Figure (1) & (2).

Then \( Y(f) \) is
(a) \(-\frac{1}{2}X(f/2)e^{-j2\pi f}\)  \( c) -X(f/2)e^{j2\pi f} \)
(b) \(-\frac{1}{2}X(f/2)e^{j2\pi f}\)  \( d) -X(f/2)e^{-j2\pi f} \)

[GATE 2004: 2 Marks]
Soln. The figures of $x(t)$ and $y(t)$ are given, from these figures.

\[ y(t) = -x(2t + 2) \]

If $x(t) \leftrightarrow X(f)$

Then $x(t + 2) \rightarrow e^{j2\pi tf} X(f)$

Using time shifting property

\[ x(2t + 2) \rightarrow \left(\frac{1}{2}\right) X(f/2)e^{j2\pi f} \]

According to time scaling property

\[ y(f) = -\frac{1}{2}X(f/2)e^{j2\pi f} \]

Option (b)

15. For a signal $x(t)$ the Fourier transform is $X(f)$. Then the inverse Fourier transform of $X(3f + 2)$ is given by

(a) $\frac{1}{2}x\left(t\right)e^{j3\pi t}$

(b) $\frac{1}{3}x\left(t\right)e^{-j4\pi t/3}$

(c) $3x(3t)e^{-j4\pi t}$

(d) $x(3t + 2)$

[\text{GATE 2005: 2 Marks}]

Soln. In this problem we use the following two properties of Fourier Transform

If $x(t) \rightarrow X(f)$

\[ e^{\pm j2\pi ft} x(t) \longrightarrow X(f \pm f_0) \hspace{1cm} (1) \]

Frequency shifting property

\[ \frac{1}{|k|} x\left(\frac{t}{k}\right) \longrightarrow (kf) \hspace{1cm} (2) \]

Time scaling property

Using frequency shift property

\[ e^{-j4\pi t} x(t) \longrightarrow X(f + 2) \]

Using time scaling property
\[ \frac{1}{3} x \left( \frac{t}{3} \right) e^{-4\pi t/3} \rightarrow X(3f + 2) \]

Option (b)

16. Two of the angular frequencies at which its Fourier transform becomes zero are
(a) \( \pi, 2\pi \)
(b) \( 0.5\pi, 1.5\pi \)
(c) \( 0, \pi \)
(d) \( 2\pi, 2.5\pi \)

\[ \text{[GATE 2008: 2 Marks]} \]

Soln. The given time function \( x(t) \) is shown in figure

![Time function graph]

Its Fourier Transform \( X(f) \) is given by
\[ X(f) = 2 \sin c(2f) \]
\[ = 2 \text{ for } f = 0 \]
\[ = 0 \text{ for } 2f = \pm 1, \pm 2, \ldots \]

Or \( \omega = 2\pi f = \pm \pi, \pm 2\pi, \pm 3\pi, \ldots \)

Option (a)

17. The Fourier transform of a signal \( h(t) \) is \( H(j\omega) = (2 \cos \omega)(\sin 2\omega)/\omega \).

The value of \( h(0) \) is
(a) \( 1/4 \)
(b) \( 1/2 \)
(c) \( 1 \)
(d) \( 2 \)

\[ \text{[GATE 2012: 2 Marks]} \]
Soln.

\[ H(j\omega) = \frac{(2\cos \omega)(\sin 2\omega)}{\omega} \]

\[ = \frac{2\sin 2\omega \cdot \cos \omega}{\omega} \]

\[ = \frac{\sin 3\omega + \sin \omega}{\omega} \]

\[ = \frac{\sin 3\omega}{\omega} + \frac{\sin \omega}{\omega} \]

So, inverse Fourier Transform of \( H(j\omega) \)

\[ h(t) = h_1(t) + h_2(t) \]

\[ h(0) = h_1(0) + h_2(0) = \frac{1}{2} + \frac{1}{2} = 1 \]

Option (c)
18. Let \( g(t) = e^{-\pi t^2} \), and \( h(t) \) is filter matched to \( g(t) \). If \( g(t) \) is applied as input to \( h(t) \), then the Fourier transform of the output is
(a) \( e^{-\pi t^2} \)
(b) \( e^{-\pi f^2 / 2} \)
(c) \( e^{-\pi |f|} \)
(d) \( e^{-2\pi f^2} \)

[GATE 2013: 1 Mark]

Soln. Given, \( g(t) = e^{-\pi t^2} \)

\( h(t) \) is matched to \( g(t) \)

\[
\begin{align*}
\text{LTI System} & \quad g(t) \quad \rightarrow \quad H(f) \quad \rightarrow \quad y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) \, d\tau \\
\text{LTI System} & \quad G(f) \quad \rightarrow \quad H(f) \quad \rightarrow \quad y(f) = X(f) \cdot H(f)
\end{align*}
\]

\( g(t) = e^{-\pi t^2} \) (Gaussian Pulse)

\( G(f) = e^{-\pi f^2} \) (Fourier Transform of Gaussian Pulse)

\( h(f) = e^{-\pi f^2} \) (Since filter is matched)

\( y(f) = G(f) \cdot h(f) = e^{-\pi f^2} \cdot e^{-\pi f^2} \)

\( y(f) = e^{-2\pi f^2} \)

Option (d)
19. The value of the integral
\[ \int_{-\infty}^{\infty} \sin \frac{c^2}{c}(dt) \] is ________.

[GATE 2014: 1 Mark]

Soln. The given integral gives the energy of the signal \( \sin c \,(5t) \)

\[ \sin c \,(5t) = \frac{\sin 5\pi t}{5\pi t} \]

Using Perceval’s theorem

\[ E_f = \int_{-\infty}^{\infty} |f(t)|^2 \, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(\omega)|^2 \, d\omega \]

\[ Energy = \frac{1}{2\pi} \int_{-5\pi}^{5\pi} (1/5)^2 \, d\omega \]

\[ = \frac{1}{50\pi} (10\pi) = \frac{1}{5} = 0.2 \]

Answer 0.2