Basics of Control Systems

1. Tachometer feedback in a d.c. position control system enhances stability (T/F)

[GATE 1994: 1 Mark]

Soln. The tachometer feedback is a derivative feedback. Thus it adds zero at origin. Hence stability is improved

2. The transfer function of a linear system is the
   (a) ratio of the output, $V_0(t)$ and input $V_i(t)$.
   (b) ratio of the derivatives of the output and the input.
   (c) ratio of the Laplace transform of the output and that of the input with all initial conditions zeros.
   (d) none of these

[GATE 1995: 1 Mark]

Soln. The transfer function of a linear system is the ratio of Laplace transform of the output and input with all initial conditions zero

Option (c)

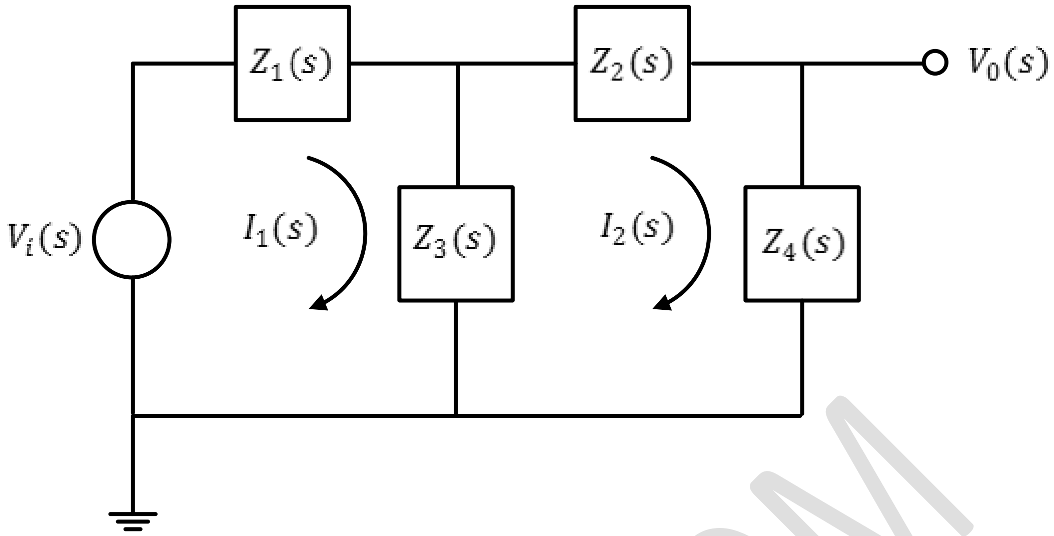
3. The transfer function of at tachometer is of the form
   (a) Ks
   (b) K/s
   (c) K/(S+1)
   (d) K/S(s+1)

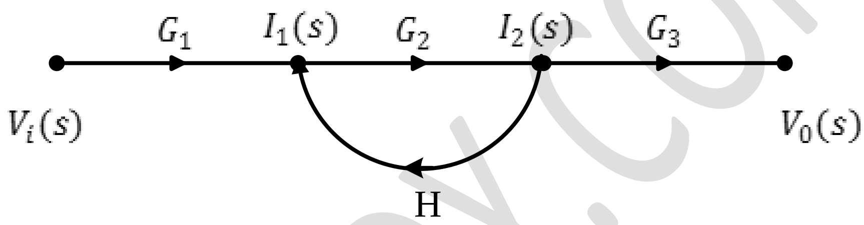
[GATE 1998: 1 Mark]

Soln. The transfer function of a tachometer is of the form ks, it adds zero at the origin

Option (a)

4. An electrical system and its signal-flow graph representations as shown in the figure (a) and (b) respectively. The values of G_2 and H respectively, are





(a) $\frac{Z_3(S)}{Z_2(S)+Z_3(3)+Z_4(S)}, \frac{-Z_3(S)}{Z_1(S)+Z_3(S)}$

(b) $\frac{-Z_3(S)}{Z_2(S)-Z_3(3)+Z_4(S)}, \frac{-Z_3(S)}{Z_1(S)+Z_3(S)}$

(c) $\frac{Z_3(S)}{Z_2(S)+Z_3(3)+Z_4(S)}, \frac{Z_3(S)}{Z_1(S)+Z_3(S)}$

(d) $\frac{-Z_3(S)}{Z_2(S)-Z_3(3)+Z_4(S)}, \frac{Z_3(S)}{Z_1(S)+Z_3(S)}$

[GATE 2001: 2 Marks]

Soln. The values of G_2 and H ?

$$V_1(s) = I_1(s)[Z_1(s) + Z_3(s)] - I_2(s)\, Z_3(3)$$

$$\frac{V_1(s)}{Z_1(s) + Z_3(s)} = I_1(s) - \frac{I_2(s)Z_3(s)}{Z_1(s) + Z_3(3)} \qquad ------(I)$$

$$\textit{In second loop}: \quad [I_2(s) - I_1(s)]\, Z_3(s) + I_2(s)\, [Z_2(s) + Z_4(s)] = 0$$

$$or \quad I_2(s)[Z_2(s) + Z_3(s) + Z_4(s)] = I_1(s)\, Z_3(s)$$

$$G_2 = \frac{I_2(s)}{I_1(s)} = \frac{Z_3(s)}{Z_2(s) + Z_3(s) + Z_4(s)} \quad --------- (II)$$

$$From \quad SFG \quad V_1 G_1(s) + I_2(s)\, H(s) = I_1(s)$$

$$V_1 G_1(s) = I_1(s) - I_2(s)\, H(s)$$

Comparing with ---------(I)

$$G_1(s) = \frac{1}{Z_1(z) + Z_3(s)}\ , \qquad H(s) = \frac{Z_3(s)}{Z_1(s) + Z_3(s)}$$

Option (c)

5. The open-loop DC gain of a unity negative feedback system with closed-loop transfer
Function $\frac{S+4}{S^2+7S+13}$ is
(a) 4/13 (c) 4
(b) 4/9 (d) 13

[GATE 2001: 2 Marks]

Soln. Closed loop transfer function

$$= \frac{G(s)}{1 + G(s)H(s)}$$

$$\frac{G(s)}{1 + G(s)\, H(s)} = \frac{s+4}{s^2 + 7s + 13}$$

$$\frac{1 + G(s)\, H(s)}{G(s)} = \frac{s^2 + 7s + 13}{s+4}$$

$$H(s) = 1 \quad for\ unity\ feedback$$

$$\frac{1}{G(s)} = \frac{s+4}{s^2 + 7s + 13} - 1$$

$$= \frac{s^2 + 6s + 9}{s + 4}$$

$$G(s) = \frac{s + 4}{s^2 + 6s + 9}$$

for DC, s = 0

$$G(s) = \frac{4}{9}$$

Option (b)

6. A system described by the following differential equation $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = x(t)$ is initially at rest. For input $x(t) = 2u(t)$, the output y(t) is
(a) $(1 - 2e^{-t} + e^{-2t})u(t)$
(b) $(1 + 2e^{-t} - 2e^{-2t})u(t)$
(c) $(0.5 + e^{-t} + 1.5e^{-2t})u(t)$
(d) $(0.5 + 2e^{-t} + 2e^{-2t})u(t)$

[GATE 2004: 2 Marks]

Soln.

$$\frac{d^2y}{dt^2} + \frac{3dy}{dt} + 2y = x(t)$$

$X(t) = 2u(t)$

Taking Laplace Transform

$s^2y(s) + 3s\, y(s) + 2y(s) = X(s)$

$$(s^2 + 3s + 2)\, y(s) = \frac{2}{s}$$

$$y(s) = \frac{2}{s(s+2)(s+1)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+1} = \frac{1}{s} + \frac{1}{s+2} - \frac{2}{s+1}$$

$y(t) = [1 + e^{-2t} - 2e^{-t}]u(t)$

Option (a)

7. In the system shown below, $x(t) = (\sin t)u(t)$. In steady-state, the response $y(t)$ will be

(a) $\frac{1}{\sqrt{2}} sin\left(t - \frac{\pi}{4}\right)$

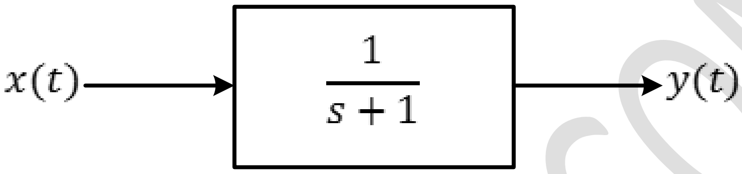
(b) $\frac{1}{\sqrt{2}} sin\left(t + \frac{\pi}{4}\right)$

(c) $\frac{1}{\sqrt{2}} e^{-t} \sin t$

(d) $\sin t - \cos t$

[GATE 2006: 1 Mark]

Soln.



$$x(t) = \sin t \ u(t)$$

$$\omega = 1\, rad/sec$$

$$y(t) = x(t) * h(t)$$

$$y(s) = x(s)\, H(s)$$

$$H(s) = \frac{1}{s+1}$$

$$H(j\omega) = \frac{1}{j+1} = \frac{1}{\sqrt{2}}\ tan^{-1} \angle -45^0 \quad as \quad \omega = 1\, rad/sec$$

$$x(t) = \sin t \ u(t)$$

$$y(t) = \frac{1}{\sqrt{2}} \sin\left(t - \frac{\pi}{4}\right)$$

Option (a)

8. The unit-step response of a system starting from rest is given by

$c(t) = 1 - e^{-2t}$ for $t \geq 0$

The transfer function of the system is

(a) $\frac{1}{1+2s}$

(b) $\frac{2}{2+s}$

(c) $\frac{1}{2+s}$

(d) $\frac{2s}{1+2s}$

[GATE 2006: 2 Marks]

Soln. The unit step response of a system starting from rest

$c(t) = 1 - e^{-2t}$ for $t \geq 0$

$$c(s) = \frac{1}{s} - \frac{1}{s+2}$$

$$= \frac{2}{s(s+2)}$$

$Input \rightarrow unitstep = \frac{1}{s}$

$Input \;\; X\, H(s) = c(s)$

$$\frac{1}{s}\, H(s) = \frac{2}{s(s+2)}$$

$$H(s) = \frac{2}{s(s+2)} \times \frac{s}{1}$$

$$= \frac{2}{s+2}$$

Option (b)

10. The unit impulse response of a system is $h(t) = e^{-t}, t \geq 0$

For this system, the steady-state value of the output for unit step input is equal to

(a) -1

(b) 0

(c) 1

(d) ∞

[GATE 2006: 2 Marks]

Soln. The unit impulse response of a system is $h(t) = e^{-t}, \quad t \geq 0$

The steady state value of the output for unit step input

$H(s) = \frac{1}{s+1}$

$X(s) = \frac{1}{s}$

Output $y(s) = X(s)\, H(s)$

$$= \frac{1}{s(s+1)}$$

$$y(s) = \frac{1}{s} - \frac{1}{s+1}$$

$$y(t) = (1 - e^{-t})$$

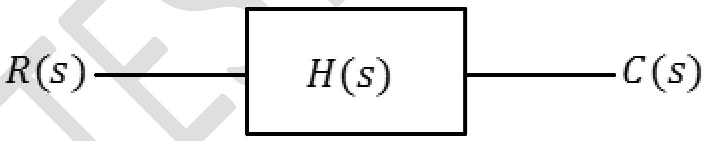
When $t \to \infty$ $(stady\ state), \quad output = 1$

Option (c)

11. A linear, time-invariant, causal continuous time system has a rational transfer function with simple poles at s = -2 and s = -4, and one simple zero at s = -1. A unit step u(t) is applied at the input of the system. At steady state, the output has constant value of 1.
The impulse response of this system is
(a) $[exp(-2t) + \exp(-4t)]\ u(t)$
(b) $[-4\exp(-2t) + 12\exp(-4t) - \exp(-t)]u(t)$
(c) $[-4\exp(-2t) + 12\exp(-4t)]u(t)$
(d) $[-0.5\exp(-2t) + 1.5\exp(-4t)]u(t)$

[GATE 2008: 2 Marks]

Soln. Transfer function



$$H(s) = \frac{K(s+1)}{(s+2)(s+4)}$$

Input $R(s) = \frac{1}{s}$

Output $c(s) = R(s)\ H(s)$

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} SF(s) = 1$$

$$or \quad \lim_{s\to 0} SC\,(s) = 1 \quad , \quad C(s) = \frac{k(s+1)}{s(s+2)(s+4)}$$

$$\lim_{s\to 0} \frac{SK(s+1)}{s(s+2)(s+4)} = 1$$

$\frac{k}{8} = 1$

$k = 8$

$$H(s) = \frac{8(s+1)}{(s+2)(s+4)}$$

$$= \frac{-4}{(s+2)} + \frac{12}{(s+4)}$$

$$h(t) = (-4e^{-2t} + 12e^{-4t})ut$$

which is the impulse response of the system

Option (c)

12. A system with the transfer function $\frac{Y(S)}{X(S)} = \frac{S}{S+P}$ has an output $y(t) = \cos\left(2t - \frac{\pi}{3}\right)$ for the input signal $x(t) = p\cos\left(2t - \frac{\pi}{2}\right)$. Then, the system parameter 'p' is

(a) $\sqrt{3}$

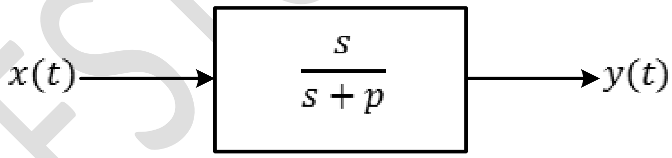
(b) $\frac{2}{\sqrt{3}}$

(c) 1

(d) $\frac{\sqrt{3}}{2}$

[GATE 2010: 1 Mark]

Soln.



$$\frac{y(s)}{x(s)} = \frac{s}{s+p} = \frac{j\omega}{j\omega+P}, \quad \phi = 90^0 - tan^{-1}\frac{\omega}{p}$$

$$y(t) = \cos\left(2t - \frac{\pi}{3}\right) \qquad \omega = 2\,rad/sec$$

$$x(t) = p\cos\left(2t - \frac{\pi}{2}\right)$$

Phase difference between input and output

$$\phi = \frac{-\pi}{3} - \left(\frac{-\pi}{2}\right)$$

$$= \frac{\pi}{2} - \frac{\pi}{3}$$

$$90 - 60 = 30^0$$

For the transfer function $\phi = 90^0 - tan^{-1}\frac{\omega}{p}$

$$90 - tan^{-1}\frac{\omega}{p} = 30^0$$

$$tan^{-1}\frac{\omega}{p} = 60$$

$$tan^{-1}\frac{2}{p} = 60^0$$

$$\frac{2}{p} = \sqrt{\frac{3}{1}}$$

$P = \frac{2}{\sqrt{3}}$ Option (c)