LTI Systems (Continuous & Discrete) - Basics

1. A system with an input x(t) and output y(t) is described by the relation: y(t) = t. x(t). This system is
   (a) linear and time-invariant
   (b) linear and time-varying
   (c) non-linear & time-invariant
   (d) non-linear and time-varying

[GATE 2000: 1 Mark]

Soln. Systems that are linear and time invariant are called LTI systems.

First check for linearity i.e. super position applies

Input output equation is

$$\boldsymbol{y(t) = t.x(t)}$$

$$\boldsymbol{a.y_1(t) = at.x_1(t)}$$

$$\boldsymbol{a.y_2(t) = at.x_2(t)}$$

So, $\boldsymbol{a[y_1(t) + y_2(t)] = a[t\,x_1(t) + tx_2(t)]}$

So, system is linear.

Check for time variance

If input is delayed then

$$\boldsymbol{y_d(t - t_0) = t\,x(t - t_0)}$$

If output is delayed then

$$\boldsymbol{y(t - t_0) = (t - t_0)\,x(t - t_0)}$$

Both are not equal, so system is time varying.

Alternative:

Since $\boldsymbol{x(t)}$ is multiplied by t, the function of time, so system is time varying.

Option (b)

2. Let δ(t) denote the delta function. The value of the integral

$$\int_{-\infty}^{\infty} \delta(t)\, cos\left(\frac{3t}{2}\right) dt \;\; is$$

(a) 1
(b) -1
(c) 0
(d) π/2

[GATE 2001: 1 Mark]

Soln. $\boldsymbol{\delta(t)}$ **is unit impulse function also known as Dirac Delta Function:**

Defined as

$$\delta(t) = \begin{cases} 0 & for\ t \neq 0 \\ \infty & for\ t = 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

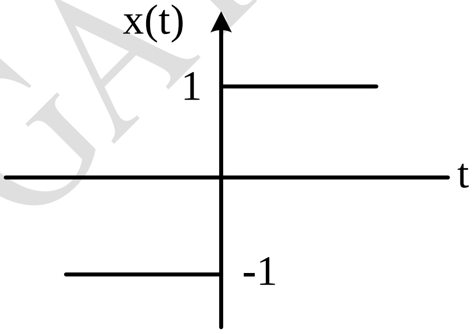
$$\delta(t)\,.\,x(t) = x(0)\,.\,\delta(t)$$

$$Here, \quad x(t) = \cos\left(\frac{3t}{2}\right) \quad then \quad x(0) = \cos(0) = 1$$

$$\int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = \int_{-\infty}^{\infty} x(0)\,\delta(t)\,dt = \int_{-\infty}^{\infty} \delta\,(t)\,dt = 1$$

Option (a)

3. The function x(t) is shown in the figure. Even and odd parts of unit-step function u(t) are respectively,



(a) $\frac{1}{2}, \frac{1}{2}x(t)$
(b) $-\frac{1}{2}, \frac{1}{2}x(t)$
(c) $\frac{1}{2}, -\frac{1}{2}x(t)$
(d) $-\frac{1}{2}, -\frac{1}{2}x(t)$

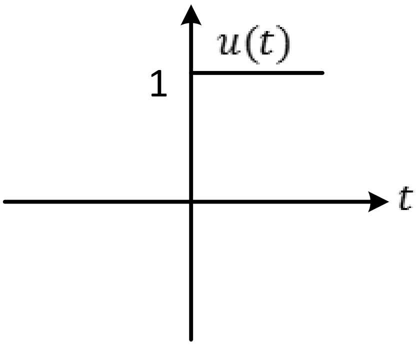
[GATE 2005: 1 Mark]

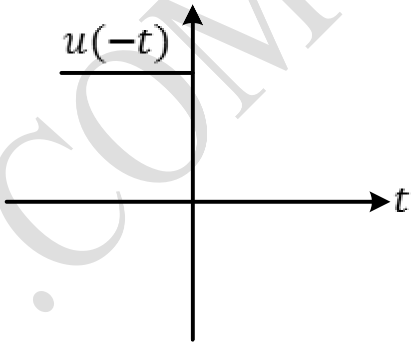
Soln. Even and odd parts of signal are given by

$$\boldsymbol{Even\ part} = \frac{\boldsymbol{\alpha(t)+\alpha(-t)}}{\boldsymbol{2}}$$

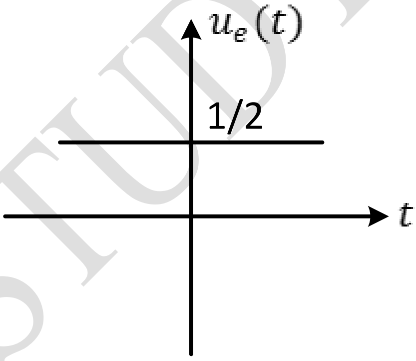
$$\boldsymbol{odd\ part} = \frac{\boldsymbol{\alpha(t)-\alpha(-t)}}{\boldsymbol{2}}$$

Let, $\boldsymbol{\alpha(t) = u(t)}$

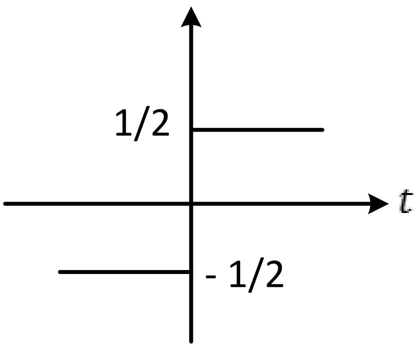




$u_e(t) = \frac{1}{2}$



$u_0(t) = \frac{x(t)}{2}$



Option (a)

4. The Dirac delta function δ(t) is defined as

   (a) $\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & otherwise \end{cases}$

   (b) $\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & otherwise \end{cases}$ and $\int_{-\infty}^{\infty} \delta(t) dt = 1$

   (c) $\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & otherwise \end{cases}$ and $\int_{-\infty}^{\infty} \delta(t) dt = 1$

   (d) $\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & otherwise \end{cases}$ and $\int_{-\infty}^{\infty} \delta(t) dt = 1$

[GATE 2006: 1 Mark]

Soln. Dirac delta function is also called unit impulse function.

It is defined as

$$\boldsymbol{\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & otherwise \end{cases} \quad and \quad \int_{-\infty}^{\infty} \delta(t) dt = 1}$$

Option (d)

5. The input and output of a continuous time system are respectively denoted by x(t) and y(t). Which of the following descriptions correspond to a casual system?

   (a) $y(t) = x(t-2) + x(t+4)$
   (b) $y(t) = (t-4)\,x(t+1)$
   (c) $y(t) = (t+4)\,x(t-1)$
   (d) $y(t) = (t+5)\,x(t+5)$

[GATE 2008: 1 Mark]

Soln. A casual system has the output that depends only on the present and past values of the input.

Expressions (a), (b) and (d) involve terms that require future values like $\boldsymbol{x(t+4), x(t+1)}$ *and* $\boldsymbol{x(t+5)}$

So, Option (c)

6. A discrete-time signal $x[n] = \sin(\pi^2 n)$, n being an integer, is
   (a) Periodic with period π
   (b) Periodic with period π^2
   (c) Periodic with period π/2
   (d) Not periodic

[GATE 2014: 1 Mark]

Soln. Discrete time signal is given

$$x[n] = \sin(\pi^2 n)$$

$$So,\ \ \omega_0 = \pi^2$$

$$So,\ N = \frac{2\pi}{\omega_0}.m \qquad (Note)\quad T = \frac{2\pi}{\omega_0}$$

Where m is the smallest integer that converts $\frac{2\pi}{\omega_0}$ into integer value

$$So,\ N = \frac{2\pi}{\pi^2}.m = \frac{2}{\pi}m$$

Thus there is no integer value of m which could make N integer.

So, the system is not periodic

Option (d)

7. Let h(t) be the impulse response of a linear time invariant system. Then the response of the system for any input u(t) is

   (a) $\int_0^t h(\tau)\,u(t-\tau)d\tau$
   (b) $\frac{d}{dt}\int_0^t h(\tau)\,u(t-\tau)d\tau$
   (c) $\int_0^t \left|\int_0^t h(\tau)\,u(t-\tau)d\tau\right| dt$
   (d) $\int_0^t h^2(\tau)\,u(t-\tau)d\tau$

[GATE 1995: 1 Mark]

Soln. For LTI System $h(t)$ is the impulse response find the response for any input $u(t)$

$$y(t) = u(t) * h(t)$$

$$= \int_{-\infty}^{\infty} h(\tau)\ u(t-\tau)\ d\tau$$

$$since \quad u(t) = 0 \ \ for\ t < 0 \quad (unit\ step)$$

$$So, \ \ y(t) = \int_{0}^{t} h(\tau)\, u(t-\tau)\ d\tau$$

Option (a)

8. The unit impulse response of a linear time invariant system is the unit step function u(t). For t > 0, the response of the system to an excitation $e^{-at}\ u(t), a > 0$ will be
   (a) $a\ e^{-at}$
   (b) $(1/a)\ (1 - e^{-at})$
   (c) $a(1 - e^{-at})$
   (d) $1 - e^{-at}$

[GATE 1998: 1 Mark]

Soln. $$h(t) = u(t)$$

$$H(s) = \frac{1}{s}$$

$$x(t) = e^{-at}u(t) \ \ for\ \ a > 0$$

$$Y(s) = X(s)H(s)$$

$$X(s) = \mathcal{L}[x(t)] = \frac{1}{(s+a)}$$

$$Y(s) = \frac{1}{(s+a)} \cdot \frac{1}{s} = \frac{1}{a}\left[\frac{1}{s} - \frac{1}{s+a}\right]$$

Taking inverse Laplace transform

$$y(t) = \frac{1}{a}[1 - e^{-at}]$$

Option (b)

9. Convolution of $x(t+5)$ with impulse function $\delta(t-7)$is equal to
   (a) $x(t-12)$
   (b) $x(t+12)$
   (c) $x(t-2)$
   (d) $x(t+2)$

[GATE 2002: 1 Mark]

Soln. If $x(t) * h(t) = g(t)$

Then $x(t-\tau_1) * h(t-\tau_2) = y(t-\tau_1-\tau_2)$

$$x(t+5) * \delta(t-7) = x(t+5-7) = x(t-2)$$

Option (c)

10. The impulse response $h[n]$ of a linear time-invariant system is given by $h[n] = u[n+3] + u[n-2] - 2u[n-7]$ where $u[n]$ is the unit step sequence. The above system is

(a) stable but not causal
(b) stable and causal
(c) causal but unstable
(d) unstable and not causal

[GATE 2004: 1 Mark]

Soln. **LTI System has impulse response**

$$h(n) = u[n+3] + u[n-2] - 2u[n-7]$$

where $u[n]$ is unit step sequence. The given impulse response can be written using the summation form

$$\sum_{n=-\infty}^{\infty} h(n) = \sum_{n=-3}^{\infty} u(n+3) + \sum_{n=2}^{\infty} u(n-2) - 2\sum_{n=7}^{\infty} u(n-7)$$

Due to the third term, after $n = 7$ the negative impulse will cancel with first two terms

$$So, \sum_{n=-\infty}^{\infty} h(n) = \sum_{n=-3}^{6} 1 + \sum_{n=2}^{6} 1 = 10 + 5 = 15$$

For the system to be stable

$$\int_{-\infty}^{\infty} h(t)\ dt < \infty, \quad here \quad it\ is\ 15\ \ which\ is\ finite$$

Note while summing, include $n = 0$ term also

For bounded input, the output is bounded. So the system is stable.

In the impulse response term note that future value of input i.e. $u(n+3)$

So system is <u>not causal.</u>

Option (a)

11. The impulse response h(t) of a linear time-invariant continuous time system is described by $h(t) = \exp(\alpha t)\,u(t) + \exp(\beta t)\,u(-t)$ where u(t) denotes the unit step function, and α and β are real constants. This system is stable if
(a) α is positive and β is positive
(b) α is negative and β is negative
(c) α is positive and β is negative
(d) α is negative and β is positive

[GATE 2008: 1 Mark]

Soln. An LTI continuous time system is stable if and only if its impulse response in absolutely integrable

$$i.e. \quad \int_{-\infty}^{\infty} |h(\tau)|\, d\tau < \infty$$

Given

$$h(t) = e^{\alpha t}\mu(t) + e^{\beta t}u(-t)$$

For stability

$$\alpha t < 0 \;\; for \;\; t > 0$$

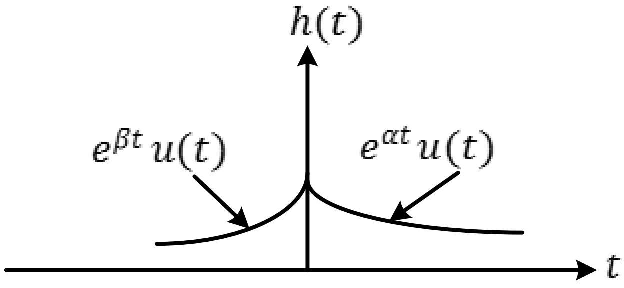
$$i.e. \;\; \alpha < 0$$

& $\beta t > 0 \;\; for \;\; t < 0$

$$i.e. \quad \beta > 0$$

Thus $\alpha \; is \; -ve \;\; and \;\; \beta$ **is +ve**

So, $h(t)$ **should be of the form shown in the figure.**



12. A system is defined by its impulse response $h(n) = 2^n u(n-2)$. The system is
(a) stable and causal
(b) causal but not stable
(c) stable but not causal
(d) unstable and non-causal

[GATE 2011: 1 Mark]

Soln. A system is defined by its impulse response

$$\boldsymbol{h(n) = 2^n u(n-2)}$$

For causal system

$$\boldsymbol{h(n) = 0 \quad for \quad n < 0}$$

Hence, the given system is causal

$$\sum_{n=2}^{\infty} 2^n = \infty \quad , \qquad \textit{So given system is not stable}$$

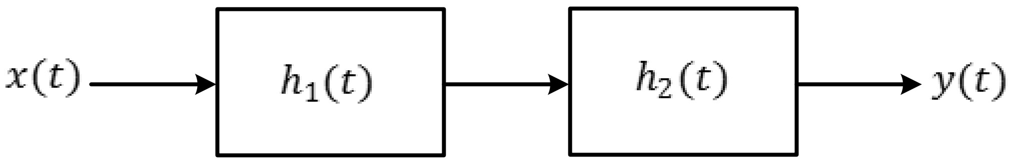
Option (b)

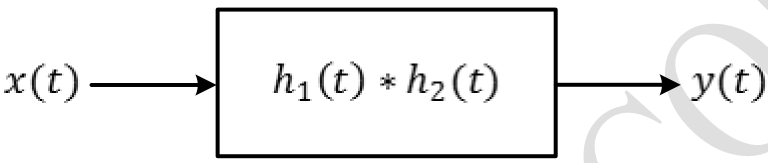
13. Two systems with impulse responses $h_1(t)$ and $h_2(t)$ are connected in cascade. Then the overall impulse response of the cascaded system is given by
(a) product of $h_1(t)$ and $h_2(t)$
(b) sum of $h_1(t)$ and $h_2(t)$
(c) convolution of $h_1(t)$ and $h_2(t)$
(d) subtraction of $h_2(t)$ from $h_1(t)$

[GATE 2013: 1 Mark]

Soln. If the two systems with impulse responses $h_1(t)$ and $h_2(t)$ are connected in cascade configuration as shown is the figure, then

overall response of the system is the convolution of the individual impulse responses.





14. Which one of following statements is NOT TRUE for a continuous time causal and stable LTI system?
   (a) All the poles of the system must lie on the left side of the jω axis.
   (b) Zeros of the system can lie anywhere in the s-plane.
   (c) All the poles must lie within $|S| = 1$.
   (d) All the roots of the characteristic equation must be located on the left side of the jω axis.

[GATE 2013: 1 Mark]

Soln. For stability of LTI Systems, all poles of system should lie in the left half of S plane and no repeated pole should be on imaginary axis

Option (c) is not true for the stable system as $|s| = 1$ have one pole in right hand plane also

Option (c)

15. A continuous, linear time has an impulse response h(t) described by

$$h(t) = \begin{cases} 3 & for\ 0 \le t3 \\ 0 & otherwise \end{cases} \qquad 0 \le t \le 3$$

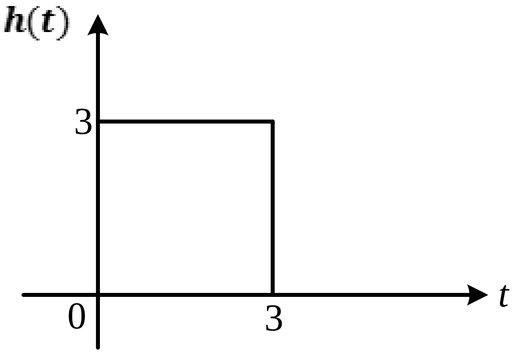
when a constant input of value 5 is applied to this filter, the steady state output is--------

[GATE 2014: 1 Mark]

Soln. Given

$$h(t) = \begin{cases} 3 \;\; for \;\; 0 \le t \le 3 \\ 0 \quad otherwise \end{cases}$$

this can be plotted as shown



$$h(t) = 3[u(t) - u(t-3)]$$

$$H(s) = 3\left[\frac{1}{s} - \frac{e^{-3s}}{s}\right]$$

Given $\quad x(t) = 5 \;\; So, \;\; X(s) = \frac{5}{s}$

We know that

$$y(s) = H(s)\; X(s)$$

$$= 3\left[\frac{1}{s} - \frac{e^{-3s}}{s}\right] . \frac{5}{s}$$

Steady state output is given by

$$\lim_{s\to 0} s.y(s)$$

$$= \lim_{s\to 0} s.\frac{15}{s}\left[\frac{1 - e^{-3s}}{s}\right]$$

$$= \lim_{s\to 0} \frac{15(1 - e^{-3s})}{s}$$

$$= 15 \lim_{s\to 0} 3e^{-3s} \qquad \text{L Hospital's rule}$$

$$= \mathbf{15 \times 3 = 45}$$

Answer : 45

16. The input $-3e^{-2t}u(t)$, where u(t) is the unit step function, is applied to a system with transfer function $\frac{s-2}{s+3}$. If the initial value of the output is -2, the value of the output at steady state is-------

[GATE 2014: 1 Mark]

Soln. Given

$$\boldsymbol{x(t) = -3e^{-2t}}$$

Find Laplace transform of $x(t)$

$$X(s) = \frac{-3}{(s+2)}$$

$$Also, \quad H(s) = \frac{s-2}{s+3}$$

$$y(s) = X(s)H(s)$$

$$y(s) = \frac{-3(s-2)}{(s+3)(s+2)}$$

Using final value theorem, the steady state value of $y(s)$ is

$$y(\infty) = \lim_{s \to 0} sy(s)$$

= 0

Answer : 0