Stability Analysis (Part – I)

1. The number of roots of
   $S^3 + 5s^2 + 7s + 3 = 0$ in the left half of the s – plane is
   (a) Zero (c) Two
   (b) One (d) Three

[GATE 1998 : 1 Mark]

Soln. Using R – H criterion

s^3	1	7
s^2	5	3
s^1	6.4	0
s^0	3	

There is no sign change in the first column of R – H array, so no roots lie in RHS of s – plane. All the three roots lie in the left half of s – plane

Option (d)

2. The open loop transfer function of an unity feedback open loop system $\frac{2s^2+6s+5}{(s+1)^2(s+2)}$. The characteristic equation of the closed loop system is
   (a) $2s^2 + 6s + 5 = 0$
   (b) $(s + 1)^2(s + 2) = 0$
   (c) $2s^2 + 6s + 5 + (s + 1)^2(s + 2) = 0$
   (d) $2s^2 + 6s + 5 - (s + 1)^2(s + 2) = 0$

[GATE 1998 : 1 Mark]

Soln. Characteristic equation

$$1 + GH = 0$$

$$1 + \frac{2\,s^2+6s+5}{(s+1)^2(s+2)} = 0$$

$$(s + 1)^2(s + 2) + 2s^2 + 6s + 5 = 0$$

Option (c)

3. The gain margin (in dB) of a system having the loop transfer function

$$G(s)\,H(s) = \frac{\sqrt{2}}{s(s+1)} \quad \text{is}$$

(a) 0
(b) 3
(c) 6
(d) ∞

[GATE 1999 : 1 Mark]

Soln. Loop transfer of function $G(s)H(s) = \frac{\sqrt{2}}{s(s+1)}$

It is a second order function, so its gain margin is infinity

Option (d)

4. The phase margin (in degrees) of a system having the loop transfer function

$$G(s)\,H(s) = \frac{2\sqrt{3}}{s(s+1)} \quad \text{is}$$

(a) 45^0
(b) -30^0
(c) 60^0
(d) 30^0

[GATE 1999 : 1 Mark]

Soln. Loop transfer function

$$G(s)\,H(s) = \frac{2\sqrt{3}}{s(s+1)}$$

Phase margin $\gamma = 180 + \phi gc$

ϕgc is the phase angle ϕ of loop transfer function at the gain cross over frequency where $\left|G(j\omega_g)\,H(j\omega_g)\right| = 1$ where ω_g is the gain cross over frequency

$$\left|\frac{2\sqrt{3}}{j\omega_g(j\omega_g+1)}\right| = 1$$

$$\frac{2\sqrt{3}}{\omega_g\left(\sqrt{1+\omega_g^2}\right)} = 1$$

$$2\sqrt{3} = \omega_g\sqrt{1+\omega_g^2}$$

$$\omega_g = \sqrt{3}$$

$$\angle G(j\omega_g)H(j\omega_g) = -90^0 - tan^{-1}\,\omega_g$$

$$= -90^0 - tan^{-1}\sqrt{3}$$

$$= -150^0$$

Phase margin $\gamma = 180 + \phi_{gc} = 180 - 150 = 30^0$

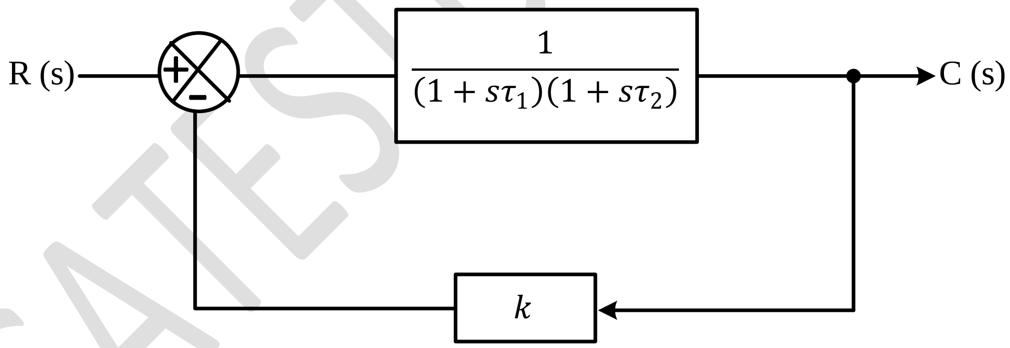
Option (d)

5. An amplifier with resistive negative feedback has two left half plane poles in its open – loop transfer function. The amplifier

(a) Will always be unstable at high frequency
(b) Will be stable for all frequency
(c) May be unstable, depending on the feedback factor
(d) Will oscillate at low frequency

[GATE 2000 : 1 Mark]

Soln. The block diagram of the system is shown below



For resistive negative feedback, the feedback factor is always less then unity. The system is stable for all frequencies

Option (b)

6. The phase margin of a system with the open – loop transfer function

$$G(s)H(s) = \frac{(1-s)}{(1+2)(2+s)}$$

(a) 0^0 (c) 90^0

(b) 63.4^0 (d) ∞

[GATE 2002 : 1 Mark]

Soln. $G(s)\,H(s) = \frac{1-s}{(1+s)(2+s)}$

Let ω_g be the gain cross over frequency where $|G(s)\,H(s)| = 1$

$$\left|\frac{1-s}{(1+s)(2+s)}\right| = \left|\frac{1-j\omega_g}{(1+j\omega_g)(2+j\omega_g)}\right| = 1$$

$$or, \quad \frac{\sqrt{1+\omega_g^2}}{\sqrt{1+\omega_g^2}\quad\sqrt{4+\omega_g^2}} = 1$$

$$or, \ \sqrt{4+\omega_g^2} = 1$$

$$\omega_g^2 = -3$$

ω_g is imaginary so no gain cross over frequency

Phase margin $\gamma = \infty$

Option (d)

7. The gain margin for the system with open – loop transfer function

$$G(s)H(s) = \frac{2(1+s)}{s^2} \quad \text{is}$$

(a) ∞ (c) 1

(b) 0 (d) $-\infty$

[GATE 2004 : 1 Mark]

Soln. $G(s)H(s) = \frac{2(1+s)}{s^2}$

The gain margin GM is the value of gain to be added to the system to bring the system to the verge of instability

$$GM = \frac{1}{|G(j\omega_{pc})\,H(j\omega_{pc})|} = \frac{1}{M}$$

ω_{pc} is the phase cross over frequency where $\angle G(s)H(s) = -180^0$

$$G(j\omega)\ H(j\omega) = \frac{2(1+j\omega)}{j\omega\ \ j\omega}$$

$$\angle G(j\omega)\ H(j\omega) = -90^0 - 90^0 + tan^{-1}\omega$$

$$= -180^0 + tan^{-1}\omega$$

$$at\ \ \omega = \omega_{pc}, \quad \phi = -180^0$$

$$-180 = -180 + tan^{-1}\omega_{pc}$$

$$tan^{-1}\omega_{pc} = 0$$

$$\omega_{pc} = 0$$

$$M = \left|\frac{2\sqrt{1+\omega^2}}{\omega^2}\right|_{\omega_{pc}} = \infty$$

$$G.M = \frac{1}{M} = \frac{1}{\infty} = 0$$

Option (b)

8. The open – loop transfer function of a unity – gain feedback control system is given by

$$G(s) = \frac{K}{(s+1)(s+2)}$$

The gain margin of the system in dB is given by

(a) 0
(b) 1
(c) 20
(d) ∞

[GATE 2006 : 1 Mark]

Soln.

$$G(s) = \frac{K}{(s+1)(s+2)}$$

It is a second order system, so it's gain margin is ∞

Option (d)

9. If the closed – loop transfer function of a control system is given as
$T(s) = \frac{s-5}{(s+2)(s+3)}$, then it is
(a) an unstable system
(b) an uncontrollable system
(c) a minimum phase system
(d) a non – minimum phase system

[GATE 2007 : 1 Mark]

Soln. The system in which one or more zeros lie in the right half of s – plane and remaining poles and zeros in the left half of s – plane is called non minimum phase system

Option (d)

10. Consider a characteristic equation given by
$s^2 + 3s^3 + 5s^2 + 6s + K + 10$
The condition for stability is
(a) K > 5
(b) – 10 < K
(c) K > – 4
(d) – 10 < K < – 4

[GATE 1988 : 2 Marks]

Soln. The characteristic equation is

$$s^4 + 3s^3 + 5s^2 + 6s + k + 10 = 0$$

Using R – H criterion

s^4	1	5	$k+10$
s^3	3	6	
s^1	3	$k+10$	0
s	$\frac{-12-3k}{3}$	0	0
s^0	$k+10$		

For stable system, all coefficients of 1st column should be positive

$\frac{-12-3k}{3} > 0 \quad or \quad -12 - 3k > 0$

$-12 > 3k$

$-4 > k$

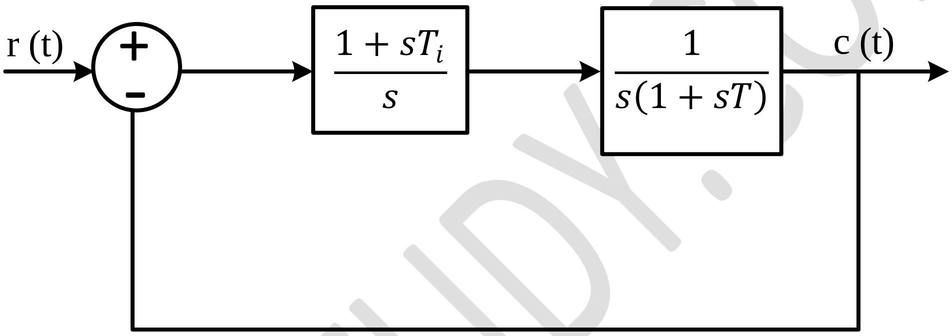
$k + 10 > 0$

$k > -10$

$-10 < k < -4$

Option (d)

11. In order to stabilize the system shown in figure. Ti should satisfy



(a) $T_i = -T$

(b) $T_i = T$

(c) $T_i < T$

(d) $T_i > T$

[GATE 1989 : 2 Marks]

Soln. Characteristic equation

$1 + GH = 0$

$1 + \frac{(1+sT_i)}{s} \times \frac{1}{s(1+sT)} \times 1 = 0$

$s^2(1 + sT) + (1 + sT_i) = 0$

$s^3T + s^2 + sT_i + 1 = 0$

Using R – H criteria

s^3	T	T_i
s^2	1	1
s^1	$T_i - T$	0
s^0	1	

For stability 1st column should be positive

$T_i - T > 0$

$T_i > T$

Option (d)

12. An electromechanical closed-loop control system has the following characteristic equation; $s^3 + 6K\,s^2 + (K+2)s + 8 = 0$. Where K is the forward gain of the system. The condition for closed loop stability is:
(a) K = 0.528
(b) K = 2
(c) K = 0
(d) K = -2.258

[GATE 1990 : 2 Marks]

Soln. $s^3 + 6ks^2 + (k+2)s + 8 = 0$

Using R – H criteria

s^3	1	$k+2$
s^2	$6k$	8
s	$\frac{6k^2+12k-8}{6k}$	0
s^0	8	

For stability, 1st column should be positive

$$\frac{6k^2+12k-8}{6k} > 0$$

$$6k^2 + 12k - 8 > 0$$

$$3k^2 + 6k - 4 > 0$$

$$k = \frac{-6 \pm \sqrt{36+48}}{6}$$

$$= \frac{-6 \pm \sqrt{84}}{6}$$

$$= \frac{-6 \pm 9.165}{6} = 0.528, -2.528$$

$$k > 0.528$$

$$k > -2.528$$

so, form the given option k = 2

Option (b)