Stability Analysis (Part – II)

1. If $s^3 + 3s^2 + 4s + A = 0$, then all the roots of this equation are in the left half plane provided that
   (a) A > 12
   (b) – 3 < A < 4
   (c) 0 < A < 12
   (d) 5 < A < 12

[GATE 1993 : 2 Marks]

Soln. $\boldsymbol{s^3 + 3s^2 + 4s + A = 0}$

Using R – H criterion

S^3	1	4
S^2	3	A
S	$\frac{12 - A}{3}$	0
S^0	A	

For stable system, 1st column should be positive.

$\boldsymbol{A > 0}$

$\boldsymbol{\frac{12-A}{3} > 0}$

or, $\boldsymbol{12 - A > 0}$

$\boldsymbol{12 > A}$

$\boldsymbol{0 < A < 12}$

Option (c)

2. If G(s) is a stable transfer function, then $F(s) = \frac{1}{G(s)}$ is always a stable transfer function. (T/F)

[GATE 1994 : 2 Marks]

Soln. **G(s) is a stable function** $= \frac{(s+Z_1)(s+Z_2)}{(s+P_1)(s+P_2)}$

$$F(s) = \frac{1}{G(s)} = \frac{(s+P_1)(s+P_2)}{(s+Z_1)(s+Z_2)}$$

The condition for stability is that none of the poles of G(s) should be on the right half of s-plane, but zeros become pole of F(s). Therefore F(s) need not be stable

3. A system described by the transfer function $H(s) = \frac{1}{s^3+\alpha s^2+Ks+3}$ is stable. The constraints on α and k are,

(a) $\alpha > 0, \alpha K < 3$
(b) $\alpha > 0, \alpha K > 3$
(c) $\alpha < 0, \alpha K > 3$
(d) $\alpha < 0, \alpha K < 3$

[GATE 2000 : 2 Marks]

Soln. **Transfer function** $H(s) = \frac{1}{s^3+\alpha s^2+ks+3}$

Using R – H criterion

S^3	1	k
S^2	α	3
S	$\frac{\alpha k-3}{\alpha}$	0
S^0	3	

For system to be stable $\alpha > 0$

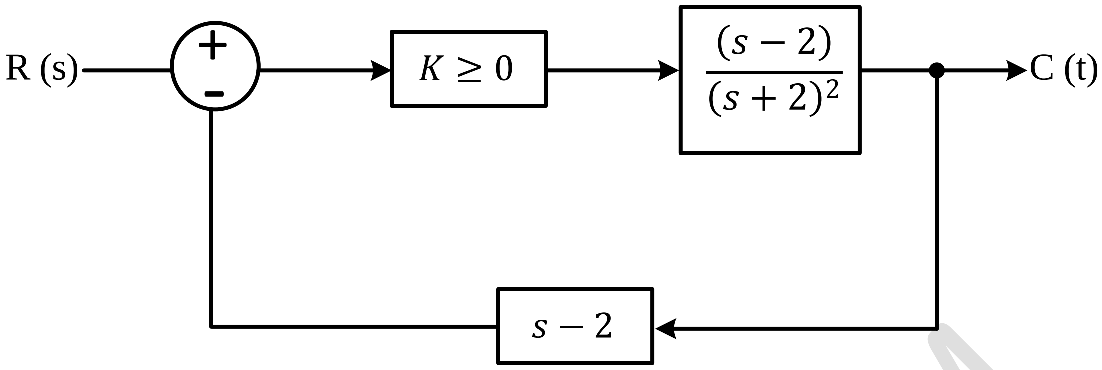
$$\frac{\alpha k-3}{\alpha} > 0$$

$$\alpha k - 3 > 0$$

$$\alpha k > 3$$

Option (b)

4. The feedback control system in the figure is stable



(a) For all $K \geq 0$

(b) only is $K \geq 0$

(c) only if $0 \leq K < 1$

(d) only if $0 \leq K \leq 1$

[GATE 2001 : 2 Marks]

Soln.

$$\boldsymbol{T.F = \frac{G_1 G_2}{1 + G_1 G_2\, H}}$$

$$\boldsymbol{= \frac{k(s-2)/(s+2)^2}{1 + \frac{k\,(s-2)\,(s-2)}{(s+2)^2}}}$$

$$\boldsymbol{= \frac{k(s-2)}{(s+2)^2 + k(s-2)^2}}$$

Characteristic equation $\boldsymbol{= s^2 + 4 + 4s + ks^2 - 4ks + 4k}$

$\boldsymbol{(1+k)s^2 + 4s - 4ks + (4+4k) = 0}$

Or, $\boldsymbol{s^2(1+k) + s(4-4k) + (4+4k) = 0}$

Using R – H criterion

S^2	$(1+k)$	$4+4k$
S	$(4-4k)$	0
S^0	$4+4k$	

For the system to stable $\boldsymbol{4 - 4k > 0}$

Or, $\boldsymbol{1 - k > 0}$

$$1 > k$$

$$\text{Or,} \quad k < 1$$

$$1 + k > 0$$

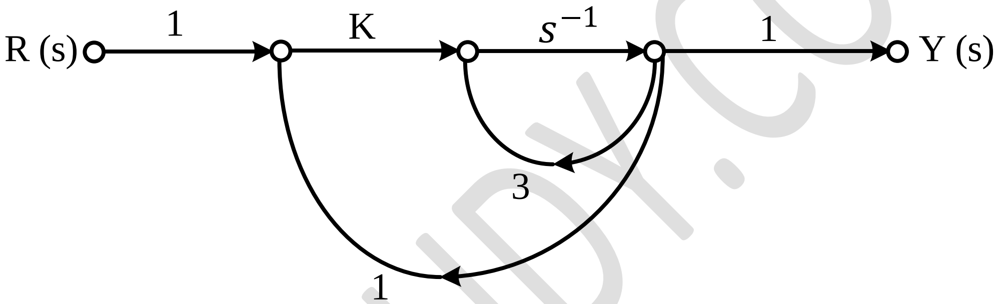
$$k > -1$$

Since it is given that $k \geq 0$ hence range of k for stability is

$$0 \leq k < 1$$

Option (c)

5. The system shown in the figure remains stable when



(a) $K < -1$
(b) $-1 < K < 1$
(c) $1 < K < 3$
(d) $K < -3$

[GATE 2002 : 2 Marks]

Soln.

$$\frac{Y(s)}{R(s)} = \frac{\frac{k}{s}}{1 - \left(\frac{3}{s} + \frac{k}{s}\right)}$$

$$= \frac{k}{s - (3 + k)}$$

For system to stable

$$3 + k < 0$$

$$k < -3$$

Option (d)

6. The characteristic polynomial of system is
$q(s) = 2s^5 + s^4 + 4s^3 + 2s^2 + 2s + 1$. The system is
(a) stable (c) unstable
(b) marginally stable (d) oscillatory

[GATE 2002 : Marks]

Soln. $q(s) = 2s^5 + s^4 + 4s^3 + 2s^2 + 2s + 1$

Routh table is

S^5	2	4	2
S^4	1	2	1
S^3	0	0	
S^2			
S^2			
S^1			
S^0			

The row with all zeros indicate the possibility of roots on imaginary axis.

The auxiliary polynomial is

$$s^4 + 2s^2 + 1 = 0$$

$$(s^2 + 1)^2 = 0$$

$$s = \pm j, s = \pm j$$

$$\frac{d}{ds}(s^4 + 2s^2 + 1) = 0$$

$$4s^3 + 4s = 0$$

$$s(4s^2 + 4) = 0$$

$$s = 0, \; s = \pm j$$

The roots are $s = 0, \; s = \pm j, \; s = \pm j$

The system is unstable, because of repeated roots on imaginary axis

Option (c)

7. The system with the open loop transfer function $G(s)H(s) = \frac{1}{s(s^2+s+1)}$ has a gain margin of

(a) – 6 dB
(b) 0 dB
(c) 3.5 dB
(d) 6 dB

[GATE 2002 : Marks]

Soln. Open loop transfer function

$$G(s)H(s) = \frac{1}{s(s^2+s+1)}$$

$$\angle G(j\omega)\, H(j\omega) = -90^0 - \tan\frac{-\omega}{1-\omega^2}$$

$$at \quad \omega = \omega_{pc}, \qquad \varphi = -180^0$$

$$so, \quad -180^0 = -90^0 - \tan\frac{-\omega_{pc}}{1-\omega_{pc}^2}$$

$$90^0 = \tan\frac{-\omega}{1-\omega_{pc}^2}$$

$$1-\omega_{pc}^2 = 0$$

$$\omega_{pc} = 1$$

Value of gain at $\omega_{pc} = 1$

$$|G(s)H(s)| = M = \frac{1}{|\omega_{pc}|\sqrt{\left(1-\omega_{pc}^2\right)^2 + \omega_{pc}^2}}$$

$$= 1$$

$$G.M = \frac{1}{M} = 1$$

$$G.M_{db} = 20log_{10}^{1} = 0$$

Option (b)

8. The open-loop transfer function of a unity feedback system is $G(s) = \frac{K}{s(s^2+s+2)(s+3)}$. The range of K for which the system is stable is

(a) $\frac{21}{4} > K > 0$

(b) $13 > K > 0$

(c) $\frac{21}{4} < K < \infty$

(d) $-6 < K < \infty$

[GATE 2004 : 2 Marks]

Soln.

$$\boldsymbol{G(s) = \frac{k}{s(s^2+s+2)(s+3)}}$$

$$\boldsymbol{H(s) = 1}$$

$$\boldsymbol{1 + G(s)H(s) = 1 + \frac{k}{s(s^2+s+2)(s+3)}}$$

$$\boldsymbol{= 1 + \frac{k}{s(s^3+3s^2+s^2+3s+2s+6)}}$$

$$\boldsymbol{= \frac{s^4+4s^3+5s^2+6s+k}{s^4+4s^3+5s^2+6s} = 0}$$

Or $\boldsymbol{s^4+4s^3+5s^2+6s+k = 0}$

S^4	1	5	k
S^3	4	6	0
S^2	$\frac{7}{2}$	k	0
S	$\frac{21-4k}{7/2}$	0	
S^0	k		

for the system to be stable $\boldsymbol{k > 0}$

$$\boldsymbol{(21-4k)\frac{2}{7} > 0}$$

$$\frac{24}{4} > k \rightarrow k < \frac{21}{4}$$

$$\frac{21}{4} > k > 0$$

Option (a)

9. For the polynomial $P(s) = s^5 + s^4 + 2s^3 + 2s^2 + 3s + 15$, the number of roots which lie in the right half of the s-plane is

(a) 4 (c) 3

(b) 2 (d) 1

[GATE 2004 : 2 Marks]

Soln. $\boldsymbol{P(s) = s^5 + s^4 + 2s^3 + 2s^2 + 3s + 15}$

$$\begin{array}{c|ccc} S^5 & 1 & 2 & 3 \\ S^4 & 1 & 2 & 15 \\ S^3 & 0 & -12 & 0 \\ S^2 & \frac{2\in+12}{\in} & 15 & \\ S & \frac{-12\left(\frac{2\in+12}{\in}\right) - 15\in}{\left(\frac{2\in+12}{\in}\right)} & & \\ S^0 & 15 & & \end{array}$$

$\in$ be a small positive number

$$\frac{\mathbf{2 \in +12}}{\in} = \pm$$

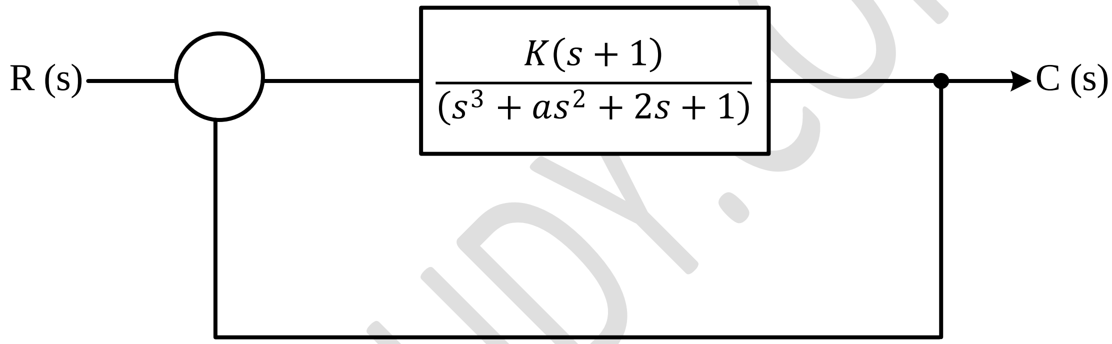
$$s = -12 - \frac{15 \in}{t}$$

$$s^0 = 15$$

The number of sing changes in first column from $s^2 to\ s$ and from $s\ to\ s^2$ are two. Two roots on right half of s – plane

Option (b)

10. The positive values of “K” and “a” so that the system shown in the figure below oscillates at a frequency of 2 rad/sec respectively are



(a) 1, 0.75
(b) 2, 0.75
(c) 1, 1
(d) 2, 2

[GATE 2006 : 2 Marks]

Soln.

$$1 + G(s)H(s) = 1 + \frac{k(s+1)}{s^3 + as^2 + 2s + 1} = 0$$

$$\frac{s^3 + as^2 + 2s + 1 + ks + k}{s^3 + as^2 + 2s + 1} = 0$$

$$s^3 + as^2 + (2+k)s + k + 1 = 0$$

S^3	1	$(2+k)$
S^2	a	$(k+1)$
S	$\frac{a(2+k)-(k+1)}{a}$	0

For oscillations,

$$\frac{a(2+k)-(k+1)}{a}=0$$

$$a=\frac{k+1}{2+k}$$

The auxiliary polynomial is

$$as^2+k+1=0$$

$$s=j\omega,\quad s^2=-\omega^2=-4 \qquad \text{given } \omega=2 \text{ rad/sec}$$

So, $-4a+k+1=0$

$$a=\frac{k+1}{4}=\frac{k+1}{k+2}$$

$$k=2,\quad a=0.75$$

Option (b)

11. Consider a unity gain feedback control system whose open loop transfer function is $G(s)=\frac{as+1}{s^2}$. The value of "a" so that the system has a phase-margin equal to $\pi/4$ is approximately equal to

(a) 2.40
(b) 1.40
(c) 0.84
(d) 0.74

[GATE 2006 : 2 Marks]

Soln.

$$G(s)=\frac{as+1}{s^2}$$

$$\angle G(j\omega)=tan^{-1}a\omega-180^0$$

Phase margin $= \frac{\pi}{4}$

$$PM = 180 + \tan^{-} a\omega - 180^0 = \frac{\pi}{4}$$

$$tan^{-} a\omega = \frac{\pi}{4}$$

$$\tan\frac{\pi}{4} = a\omega$$

$$a\omega = 1$$

Then gain crossover frequency $\omega = \omega_{gc}$ $\quad$ *where* $|G(s)| = 1$

$$\sqrt{\frac{1+a^2\omega^2}{\omega^2}} = 1 \qquad a\omega = 1$$

$$\sqrt{\frac{1+1}{\omega^2}} = 1$$

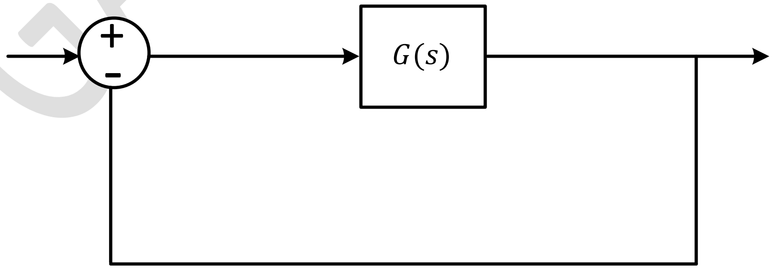
$$\omega^2 = \sqrt{2}$$

$$\omega = 2^{\frac{1}{4}}$$

$$a = \frac{1}{2^{1/4}} = 0.84$$

Option (c)

12. A certain system has transfer function $G(s) = \frac{s+8}{s^2+\alpha s-4}$, where α is parameter. Consider the standard negative unity feedback configuration as shown below



Which of the following statements is true?

(a) The closed loop system is never stable for any value of α

(b) For some positive value of α, the closed loop system is stable, but not for all positive values

(c) For all positive value of α, the closed loop system is stable

(d) The closed loop system is stable for all value of α, both positive and negative

[GATE 2008 : 2 Marks]

Soln.

$$G(s) = \frac{s+8}{s^2+\alpha s-4}$$

Closed loop gain is $\frac{G(s)}{1+G(s)}$

$$\frac{G(s)}{1+G(s)} = \frac{s+8}{s^2+\alpha s-4+s+8}$$

$$= \frac{s+8}{s^2+(\alpha+1)s+4}$$

Characteristic equation

$$q(s) = s^2+(\alpha+1)s+4$$

S^2	1	4
S	$\alpha+1$	0
S^0	4	

The closed loop system is stable for $\alpha+1>0$

$$\alpha > -1$$

For all positive value of α**, the closed loop system is stable**

Option (c)

13. The number of open right half plane poles of

$$G(s) = \frac{10}{s^5+2s^4+3s^3+6s^2+5s+3} \; is$$

(a) 0
(b) 1
(c) 2
(d) 3

[GATE 2008 : 2 Marks]

Soln.

$$G(s) = \frac{10}{s^5+2s^4+3s^3+6s^2+5s+3}$$

Using R – H criteria

S^5	1	3	5
S^4	2	6	3
S^3	$0(\in)$	$\frac{7}{2}$	
S^2	$\frac{6\in -7}{\in} = \frac{7}{\in}$	3	
S^1	$\left\{\frac{-\frac{7}{\in}\left(\frac{7}{2}\right) - 3\in}{-7/\in}\right\} + ve$		
S^0	3		

In the first column, there are two sign changes occurs hence two poles lie in the right half of s – plane

Option (c)