Frequency Response Analysis (Part - I)

1. A system has fourteen poles and two zeros. Its high frequency asymptote in its magnitude plot having a slope of:
   (a) – 40 dB/decade  (c) – 280 dB/decade
   (b) – 240 dB/decade  (d) – 320 dB/decade
   [GATE 1987: 2 Marks]

   Soln. Poles (P) = 14
   Zeros (Z) = 2
   P – Z = 14 – 2
   = 12

   \[
   \lim_{\omega \to \infty} \text{slope} = (P - Z) \left( -\frac{20\text{dB}}{\text{dec}} \right)
   \]
   \[
   = -240\text{dB/decade}
   \]
   Option (b)

2. The polar plot of \( G(s) = \frac{10}{s(s+1)^2} \) intercepts real axis at \( \omega = \omega_0 \). Then, the real part and \( \omega_0 \) are respectively given by:
   (a) – 2.5, 1  (c) – 5, 1
   (b) – 5, 0.5  (d) – 5, 2
   [GATE 1987: 2 Marks]

   Soln. \( G(s) = \frac{10}{s(s+1)^2} = \frac{10}{s(s+1)(s+1)} \)
   \[
   \angle G(j\omega) = -90^0 - 2 \tan^{-1}\omega
   \]
   \( \omega_{pc} \) is the phase cross over frequency where
   \[
   \angle G(j\omega) = -180^0
   \]
   so \( -180^0 = -90^0 - 2\tan^{-1}\omega_{pc} \)
   \[
   2\tan^{-1}\omega_{pc} = 90^0
   \]
   \( \omega_{pc} = 1 \text{ rad/sec} \)
\[ |G|_{\omega = \omega_{pc}} = \frac{10}{\omega \sqrt{1+\omega^2} \sqrt{1+\omega^2}} \]
\[ = \frac{10}{1 \times \sqrt{2} \times \sqrt{2}} \]
\[ = \frac{10}{2} = 5 \]

At \( \omega = \omega_{pc} \) the polar plot crosses the negative real axis at \(-5\)

Option (c)

3. From the Nicholas chart one can determine the following quantities pertaining to a closed loop system:
   (a) Magnitude and phase
   (b) Band width
   (c) Only magnitude
   (d) Only phase

   [GATE 1989: 2 Marks]

Soln. Nicholas chart is magnitude versus phase plot

4. The open-loop transfer function of a feedback control system is
   \[ G(s).H(s) = \frac{1}{(s + 1)^3} \]

   The gain margin of the system is
   (a) 2
   (b) 4
   (c) 8
   (d) 16

   [GATE 1991: 2 Marks]

Soln.
   \[ G(s).H(s) = \frac{1}{(s + 1)^3} \]

   \[ GM = \frac{1}{|G(j\omega_{pc})H(j\omega_{pc})|} = \frac{1}{M} \]

   \( \omega_{pc} \) is the phase cross over frequency where
   \[ \angle G(s)H(s) = -180^0 \]

   \[ G(s)H(s) = \frac{1}{(s+1)(s+1)(s+1)} \]
\[-3 \tan^{-1} \omega_p c = -180^0\]
\[\tan^{-1} \omega_p c = -60^0\]
\[\omega_p c = \sqrt{3} \text{ rad/sec}\]
\[M = |G(j\omega_p c) H(j\omega_p c)| = \frac{1}{\left(\sqrt{1+\omega_p c^2}\right)^3}\]
\[= \frac{1}{8}\]
\[GM = \frac{1}{M} = 8\]
Option (c)

5. Non-minimum phase transfer function is defined as the transfer function
   (a) which has zero in the right-half s-plane
   (b) which has zero only in the left-half s-plane
   (c) which has poles in the right-half s-plane
   (d) which has poles in the left-half s-plane

   [GATE 1995: 1 Mark]

Soln. Non minimum phase transfer function is defined as the transfer function which has one or more zeros in the right half of s – plane and remaining poles and zeros in the left half of s – plane.

   Option (a)

6. The Nyquist plot of a loop transfer function \(G(j\omega) H(j\omega)\) of a system encloses the \((-1, j0)\) point. The gain margin of the system is
   (a) less than zero
   (b) zero
   (c) greater then zero
   (d) infinity

   [GATE 1998: 1 Mark]

Soln. A system is unstable when Nyquist plot of \(G(j\omega) H(j\omega)\) enclosed the point (-1, j 0). Gain margin of unstable system is less than zero

   Option (a)
7. The Nyquist plot for the open-loop transfer function $G(s)$ of a unity negative feedback system is shown in the figure, if $G(s)$ has no pole in the right-half of $s$-plane, the number of roots of the system characteristic equation in the right-half of $s$-plane is

![Nyquist Plot](image)

(a) 0  
(b) 1  
(c) 2  
(d) 3  

[GATE 2001: 1 Mark]

Sln. $N = P - Z$

One encirclement in clockwise direction and one in anticlockwise direction house $N = 0$

Given that number of poles of $G(s)H(s)$ in the right half $s$ – plane, $p = 0$

$N = P - Z$

Or $Z = P - N$

$= 0$

So No roots of the characteristic equation or poles of the closed loop system lie in RH of $s$ – plane

Option (a)

8. In the figure, the Nyquist pole of the open-loop transfer function $G(s)H(s)$ of a system is shown. If $G(s)H(s)$ has one right-hand pole, the closed-loop system is
\[ \omega = 0 \]
\[ (-1, 0) \]

\( \omega \) positive

(a) always stable
(b) unstable with one closed-loop right hand pole
(c) unstable with two closed-loop right hand poles
(d) unstable with three closed-loop right hand poles

[**GATE 2003: 1 Mark**]

**Soln.**

\[ N = P - Z \]

The encirclement of critical point \((-1, j 0)\) is in the anticlockwise direction hence \(N = 1\)

\(P = 1\) (given)

\(Z = P - N\)

\[ = 0 \]

Hence no poles of closed loop system lie in the RH of s – plane therefore system is always stable.

Option (a)

9. A system has poles at 0.01 Hz, 1 Hz and 80 Hz; zero at 5 Hz, 100 Hz and 200 Hz. The approximate phase of the system response at 20 Hz is

(a) \(-90^0\)
(b) \(0^0\)
(c) \(90^0\)
(d) \(-180^0\)

[**GATE 2004: 2 Marks**]

**Soln.**

Phase shift are

Due to Pole at 0.01 Hz \(-90^0\)
Due to Pole at 1 Hz \(-90^0\)
Due to Pole at 80 Hz \(-90^0\)
Not to be considered as the system response at 20 Hz is to be considered
Zero at 5 Hz \(-90^0\)
Zero at 100 Hz not be considered
Zero at 200 Hz not be considered
Thus approximate total phase shift \(=-90^0 -90^0 +90^0 = -90^0\)

Option (a)

10. The Nyquist plot of \(G(j\omega)H(j\omega)\) for a closed loop control system, passed through \((-1, j0)\) point in GH plane. The gain margin of the system in dB is equal to
(a) infinite (b) greater than zero (c) less than zero (d) zero

[GATE 2006: 2 Marks]

Soln. The gain margin of system is negative i.e. less than zero
Option (c)

11. For the transfer function \(G(j\omega) = 5 + j\omega\), the corresponding Nyquist plot for positive frequency has the form

(a) \(\frac{5}{1} \) (b) \(\frac{j5}{1} \) (c) \(\frac{1}{5} \) (d) \(\frac{1}{5} \)
Soln. The transfer function $G(j\omega) = 5 + j\omega$

$$|G(j\omega)| = \sqrt{25 + \omega^2}$$

At $\omega = 0$ $|G(0)| = 5$

At $\omega = \infty$ $|G(\infty)| = \infty$

Option (a)

12. Consider the feedback system shown in the figure. The Nyquist plot of $G(s)$ is also shown. Which one of the following conclusions is correct?

- (a) $G(s)$ is an all-pass filter
- (b) $G(s)$ is strictly proper transfer function
- (c) $G(s)$ is a stable and minimum-phase transfer function
- (d) The closed-loop system is unstable for sufficiently large and positive $K$.

Soln. Nyquist plot is not enclosed critical point $(-1, j0)$, hence the system is stable. If the value of gain $K$ is increased, then intersection point moves towards $-\infty$ on the negative real axis which makes system unstable.