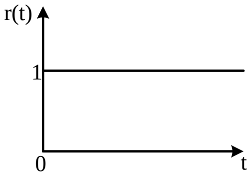
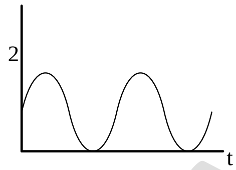
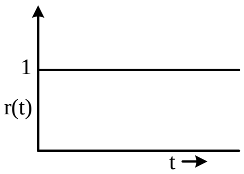
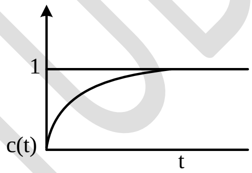
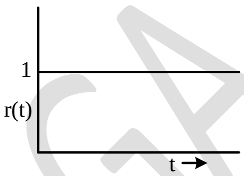
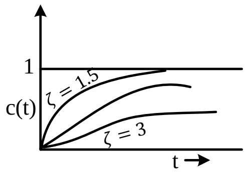
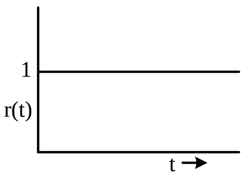
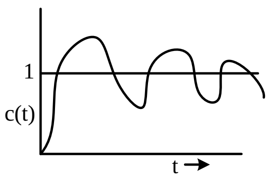
Time Response Analysis

General expression for the transfer function of II[nd] order control system is given by

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

ω_n = undamped natural frequency

$\zeta (zeta)$ = damping ratio

Type of System and it's input	Response	Roots
Case I undamped $\zeta = 0$ 	$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + \omega_n^2}$  $C(t) = 1 - \cos \omega_n t$	$S_1, S_2 = \pm j\omega_n$ The roots are purely imaginary
Case II (critically damped) $\zeta = 1$  Step input	$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2} = \frac{\omega_n^2}{(s + \omega_n)^2}$ 	$S_1, S_2 = -\omega_n$, The roots are real and equal
Case III over damped $\zeta > 1$  Step input	$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$  It taken longer time for the response to reach steady state	$S_1, S_2 = \zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$ The roots are real and unequal
Case IV underdamped $\zeta < 1$  Step input	$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$ 	$S_1, S_2 = \zeta\omega_n \pm \omega_n\sqrt{1 - \zeta^2} = -\zeta\omega_n \pm j\omega_d$ $\omega_d = \omega_n\sqrt{1 - \zeta^2}$ The roots are complex conjugate

Steady State Error:- If the actual output of a control system during steady state deviates from the reference input (i.e. desired output), the system is said to possess a steady state error.

The magnitude of the steady state error in a closed loop control system depends on it's open loop transfer function i.e. $G(s)H(s)$ of the system.

The error $E(s)$ and input $R(s)$ are related as:

$$\frac{E(s)}{R(s)} = \frac{1}{1 + G(s)H(s)}$$

$$E(s) = \frac{R(s)}{1 + G(s)H(s)}$$

Applying final value theorem,

The steady state error can be determined as

$$ess = \lim_{s \to 0} SE(s) = \lim_{s \to 0} \frac{SR(s)}{1 + G(s)H(s)}$$

Static error coefficients Kp, Kv and Ka are associated with input as unit step, unit ramp and unit parabolic, respectively.

Static positional error coefficient Kp is associated with unit step input applied to a closed loop control system

$$ess = \lim_{s \to 0} \frac{SR(s)}{1 + G(s)H(s)}$$

$$= \lim_{s \to 0} \frac{S \times \frac{1}{s}}{1 + G(s)H(s)}$$

$$= \frac{1}{1 + \lim_{s \to 0} G(s)H(s)}$$

$$= \frac{1}{1 + Kp}$$

$$Kp = \lim_{s \to 0} G(s)H(s)$$

Static Velocity error coefficient Kv. It is associated with unit ramp input applied to a closed loop control system.

Input = unit ramp

$$R(s) = \frac{1}{s^2}$$

$$ess = \lim_{s\to 0} \frac{SR(s)\ \ 1}{1 + G(s)H(s)}$$

$$= \lim_{s\to 0} \frac{s}{s^2} \frac{1}{1+G(s)H(s)}$$

$$= \lim_{s\to 0} \frac{1}{s\,\{1+G(s)H(s)\}}$$

$$= \lim_{s\to 0} \frac{1}{s\ G(s)H(s)} = \frac{1}{Kv}$$

$$Kv = \lim_{s\to 0} s\ G(s)H(s)$$

Static acceleration error coefficient Ka. It is associated with unit parabolic signal.

Input = unit parabolic

$$R(s) = \frac{1}{s^3}$$

$$ess = \lim_{s\to 0} \frac{S\ R(s)}{1+G(s)H(s)}$$

$$= \lim_{s\to 0} \frac{s}{s^3\ \{1+G(s)H(s)\}}$$

$$= \lim_{s\to 0} \frac{1}{s^2+s^2\ G(s)H(s)}$$

$$= \lim_{s\to 0} \frac{1}{s^2+s^2\ G(s)H(s)} = \frac{1}{Ka}$$

$$Ka = \lim_{s\to 0} s^2 G(s)H(s)$$