Basics of Electromagnetics – Maxwell's Equations (Part - II)

1. An electrostatic is said to be conservative when
   (a) The divergence of the field is equal to zero
   (b) The curl of the field is equal to zero
   (c) The curl of the field to $-\frac{\partial B}{\partial t}$
   (d) The Laplacian of the field is equal to $\mu\varepsilon \frac{\partial^2 E}{\partial t^2}$

[GATE 1987: 2 Marks]

Soln. An electrostatic field is said to be conservative when the closed line integral of the field is zero

$$\oint \vec{E}.\overrightarrow{dl} = 0$$

Using Stoke's theorem

$$\oint \vec{E}.\overrightarrow{dl} = \iint_S (\nabla \times \vec{E}).\overrightarrow{ds}$$

So $\nabla \times \vec{E} = 0$

Option (b)

2. On either side of a charge – free interface between two media,
   (a) the normal components of the electric field are equal
   (b) the tangential components of the electric field are equal
   (c) the normal components of the electric flux density are equal
   (d) the tangential components of the electric flux density are equal

[GATE 1988: 2 Marks]

Soln. On either side of a charge free interface between two media:

$E_{t1} = E_{t2}$ **Tangential components of electric field are equal**

$D_{n1} = D_{n2}$ **The normal components of electric flux density are equal**

Option (b) and (c)

3. Vector potential is a vector
   (a) whose curl is equal to the magnetic flux density
   (b) whose curl is equal to the electric field intensity
   (c) whose divergence is equal to the electric potential
   (d) which is equal to the vector product E x H

[GATE 1988: 2 Marks]

Soln. Vector potential A for the steady magnetic field in a homogeneous medium is related to source currents. From this magnetic potential, the magnetic field is written as:

$$\boldsymbol{\mu\vec{H} = \nabla \times \vec{A}}$$

Option (a)

4. Which of the following field equations indicate that the free magnetic charges do not exist?
   (a) $\vec{H} = \frac{1}{\mu}\nabla \times A$
   (b) $\vec{H} = \oint \frac{I\,dl \times R}{4\pi R^2}$
   (c) $\nabla . \vec{H} = 0$
   (d) $\nabla \times \vec{H} = J$

[GATE 1990: 2 Marks]

Soln. There are no isolated magnetic poles or magnetic charge on which the lines of magnetic flux can terminate. The lines of magnetic flux are continuous

$$\boldsymbol{\nabla . \vec{B} = 0}$$

In a homogenous medium $\boldsymbol{\mu}$ is independent of position so

$$\boldsymbol{\nabla . \vec{H} = 0}$$

Option (c)

5. The incoming solar radiation at a place on the surface of the earth is 1.2 KW/m^2. The amplitude of the electric field corresponding to this incident power is nearly equal to
   (a) 80 mV / m
   (b) 2.5 V / m
   (c) 30 V /m
   (d) 950 V / m

[GATE 1900: 2 Marks]

Soln. Power density P = 1.2 KW/$\boldsymbol{m^2}$, on the surface of the earth where

$$\boldsymbol{\eta = \eta_0 = (120\pi)\ ohms}$$

$$\boldsymbol{P = \frac{E^2}{2\eta}}$$

$$\boldsymbol{E = \sqrt{2\eta_o P} = \sqrt{2 \times 120\pi \times 1.2 \times 10^3} \quad v/m}$$

$$\boldsymbol{= 951\, v/m \ \cong 950\ \ v/m}$$

Option (d)

6. A long solenoid of radius R, and having N turns per unit length carries a time dependent current $I(t) = I_0 \cos(\omega t)$. The magnitude of induced electric field at a distance R/2 radially from the axis of the solenoid is
   (a) $\frac{R}{2}\mu_0\ N\ I_0\ \omega \sin(\omega t)$
   (b) $\frac{R}{4}\mu_0\ N\ I_0\ \omega \cos(\omega t)$
   (c) $\frac{R}{4}\mu_0\ N\ I_0\ \omega \sin(\omega t)$
   (d) $R\ \mu_0\ N\ I_0\ \omega \sin(\omega t)$

[GATE 1993: 2 Marks]

Soln. Number of turns per unit length = N

Current $\boldsymbol{I(t) = I_0 \cos(\omega t)}$

$$\boldsymbol{H = N\, I(t), B = \mu_0 H}$$

$$\boldsymbol{= \mu_0 N\, I(t)}$$

$$\boldsymbol{B = \mu_0\ N\ I_0 \cos(\omega t)}$$

$$\boldsymbol{\nabla \times \vec{E} = -\frac{\partial \vec{\beta}}{\partial t}}$$

Using Stoke's theorem

$$\boldsymbol{\oint \vec{E}.\vec{dl} = \int_s -\frac{\partial \beta}{\partial t}.ds}$$

$$\boldsymbol{E\,\frac{2\pi R}{2} = \mu_o\ N\ I_0 \omega \sin(\omega t)\,\frac{\pi R^2}{4}}$$

$$\boldsymbol{E = \frac{R}{4}\mu_o\ N\ I_0 \omega \sin(\omega t)}$$

Option (c)

7. Match each of the items A, B and C with an appropriate item from 1, 2, 3, 4 and 5.

(a) $\nabla \times \vec{H} = \vec{J}$

(b) $\oint_C \vec{E} \,.\, \vec{dl} = -\frac{d}{dt} \int_S \vec{B} \,.\, d\vec{s}$

(c) $\nabla . \vec{J} = -\frac{\partial \rho}{\partial t}$

(i) Continuity equation Faraday's Law
(ii) Faraday's Law
(iii) Ampere's Law
(iv) Gauss's Law
(v) Biot-Savert's Law

[GATE 1994: 2 Marks]

Soln. (A) $\boldsymbol{\nabla \times \vec{H} = \vec{J}}$

$$\oint \vec{H} \,.\, \vec{dl} = \int \vec{J} \,.\, \vec{ds} = \mathrm{I}\ \ enclosed$$

The magneto motive force around a closed path is equal to the current enclosed by the

(B)

$$\oint \vec{E} \,.\, \vec{dl} = \frac{-d}{dt} \int_S \vec{B} \,.\, \vec{ds} = \frac{-d\phi}{dt}$$

It is based on Faraday's law that the voltage around the closed path. Equal to negative of time changing magnetic flux through the surface. Enclosing the surface is

(C) $\boldsymbol{\nabla . \vec{J} = \frac{\partial \rho}{\partial F}}$

$$\oint \vec{J} \,.\, \vec{da} = -\int \frac{\partial \rho}{\partial t}$$

It is the time varying form of the equation of continuity. It states that the total current flowing out of some volume must be equal to the rate of decrease of charge with in the volume

A – 3, B – 2, C – 1

8. Two coaxial cables 1 and 2 are filled with different dielectric constants $\in_{r1}$ and $\in_{r2}$ respectively. The ratio of the wavelengths in the two cables, (λ_1/λ_2) is

(a) $\sqrt{\in_{r1}/\in_{r2}}$
(b) $\sqrt{\in_{r2}/\in_{r1}}$
(c) $\in_{r1}/\in_{r2}$
(d) $\in_{r2}/\in_{r1}$

[GATE 2000: 2 Marks]

Soln. **For a given frequency of excitation 'f ' wavelength and velocity of propagation V in a cable are related by**

$$\boldsymbol{\lambda = \frac{V}{f}}$$

$$\boldsymbol{V = \frac{1}{\sqrt{\mu\,\varepsilon}}}$$

For the same $'\mu'$ and given f

$$\lambda \propto \frac{1}{\sqrt{\in}}$$

So $$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{\epsilon_2}{\epsilon_1}} = \sqrt{\frac{\epsilon_{r2}}{\epsilon_{r2}}}$$

Option (b)

9. A material has conductivity of 10^{-2} mho/m and a relative permittivity of 4. The frequency at which the conduction current in the medium is equal to the displacement current is

(a) 45 MHz
(b) 90 MHz
(c) 450 MHz
(d) 900 MHz

[GATE 2001: 2 Marks]

Soln. **Conductivity $\boldsymbol{\sigma = 10^{-2} mho/m}$**

Relative permittivity $\boldsymbol{E_r = 4}$

Conduction current density $\boldsymbol{\overrightarrow{J_c} = \overrightarrow{\sigma E}}$

Displacement current density $\boldsymbol{\overrightarrow{J_d} = \frac{\partial \vec{D}}{\partial t}}$

$$\vec{D} = \in \vec{E}$$

For time varying filed $\vec{E} = E_0 e^{j\omega t}$

So, $\left|\vec{J_d}\right| = \omega\epsilon\vec{E}$

For $\left|\vec{J_c}\right| = \left|\vec{J_d}\right|$

$\sigma\left|\vec{E}\right| = \omega\epsilon\left|\vec{E}\right| \qquad \in_0 = \frac{1}{36\pi\times10^9}\ farads/m$

$$\sigma = \omega\epsilon$$

$$= 2\pi f\ \in_0 \in_r$$

$$f = \frac{\sigma}{2\pi\in_0 \in_r} = \frac{10^{-2}\times36\pi\times10^9}{2\pi\times4}$$

$$= 45 \times 10^6\ Hz$$

$$= 45\ MHz$$

Option (a)

10. The electric field on the surface of a perfect conductor is 2 V/m. The conductor is immersed in water $\varepsilon = 80\ \varepsilon_0$. The surface charge density on the conductor is

(a) $0\ C/\ m^2$

(b) $2\ C/\ m^2$

(c) $1.8 \times 10^{-11} C/\ m^2$

(d) $1.41 \times 10^{-9} C/\ m^2$

[GATE 2002: 2 Marks]

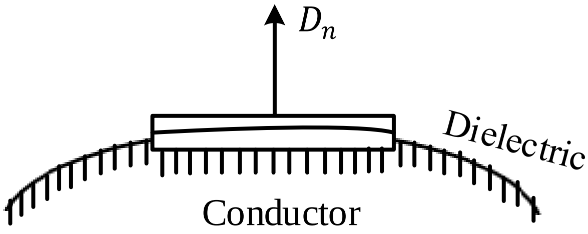
Soln. According to the boundary conditions if there is no surface charge

$$D_{n1} = D_{n2}$$

Normal component of D is continuous across the surface.

In case of metallic surface, the charge is considered to reside on surface

Appling Gauss's law to this case gives



$$D_n\, da = \rho_s\, da$$

$$or \;\; E_n = \frac{\rho_s}{\in}$$

Where da is the area of one face of the pill box and D_n is the displacement density normal to the surface

$$or \;\; E_n = \frac{\rho_s}{\in}$$

The electric displacement density at the surface of a conductor is normal to the surface of a conductor is normal to the surface and equal in magnitude to the surface charge density.

$$E_n = 2\ v/m$$

$$\rho_s = E_n \in$$

$$= E_n\, 80 \in_0$$

$$= 2 \times 80 \frac{1}{36\pi \times 10^9}$$

$$= \frac{2 \times 80 \times 10^{-9}}{36\pi}$$

$$= 1.41 \times 10^{-9}\ c/m^2$$

Option (d)

11. If the electric field intensity is given by $E = \left(X\, u_x + Y\, u_y + Z\, u_z\right)$ volt/m, the potential difference between X (2,0,0) and Y (1,2,3) is

(a) + 1volt
(b) – 1 volt
(c) + 5 volt
(d) + 6 volt

[GATE 2003: 2 Marks]

Soln. Electric field intensity,

$$E = \left(Xu_x + Yu_y + Zu_z\right) volt/m$$

The potential difference between

$$X(2,0,0) \; and \; Y(1,2,3) \quad is$$

$$V_{X_Y} = -\int_{Y(1,2,3)}^{X(2,0,0)} \vec{E}.\overrightarrow{dl}$$

$$\overrightarrow{dl} = dx \;\; u_x + dy \;\; u_y + dz \;\; u_z$$

$$\vec{E}\,.\overrightarrow{dl} = \left(Xu_x + Yu_y + Zu_z\right).\left(dx\, u_x + dy\, u_y + dz\, u_z\right)$$

$$= (X\,dx + Y\,dy + Z\,dz)$$

$$V_{XY} = -\int_{Y}^{X} (X\,dx + Y\,dy + Z\,dz)$$

$$= -\left[\int_{-1}^{2} X\,dx + \int_{2}^{0} Y\,dy + \int_{3}^{0} Z\,dz\right]$$

$$-\left[\;\left[\frac{X^2}{2}\right]_{-1}^{2} + \left[\frac{Y^2}{2}\right]_{2}^{0} + \left[\frac{Z^2}{2}\right]_{3}^{0}\;\right]$$

$$-\left[\left(\frac{4-1}{2}\right) + \left(\frac{0-4}{2}\right) + \left(\frac{0-9}{2}\right)\right]$$

$$= -\left[\frac{3-4-9}{2}\right] = 5\; volts$$

Option (c)

12. A parallel plate air-filled capacitor has plate area of 10^{-4} m^2 and plate separation of 10^{-3} m. It is connected to a 0.5 V, 3.6 GHz source. The magnitude of the displacement current is ($\varepsilon_0 = 1/36\pi \times 10^{-9}\; F/m$)

(a) 10 mA (c) 10 A
(b) 100 mA (d) 1.59 mA

[GATE 2003: 2 Marks]

Soln. Area of air filled plate – capacitor

$$A = 10^{-4}\ m^2$$

Plate separation $d = 10^{-3}\ m$

Displacement current density $\vec{J}_d = \frac{\partial \vec{D}}{\partial t}$

$\vec{J}_d = \frac{\partial \vec{D}}{\partial t}, \vec{D} = \in \vec{E},$ **for time varying field** $\vec{E} = E_0 e^{j\omega t}$

$$|J_d| = \omega \in |E|$$

$$\omega = 2\pi f, \in = \in_0 \in_r, \in_r = 1$$

$$|E| = \frac{V_s}{d}$$

$$= \frac{0.5}{10^{-3}} = \frac{10^3}{2}$$

$$|J_d| = \omega \in |E|$$

$$= 2\pi f\ \in_0\ \frac{V_s}{d}$$

$$= 2\pi \times 3.6 \times 10^9 \frac{10^{-9}}{36\pi} \times \frac{10^3}{2}$$

$$= \frac{2\pi}{10\pi} \times \frac{10^3}{2}$$

$$= 100\ A/m^2$$

Displacement current $I_d = |J_d| \times A$

$$= 100 \times 10^{-4}$$

$$= 10^{-2}\ amps$$

= 10 ma

Option (a)

13. If $\vec{E} = (\hat{a}_x + j\hat{a}_y)e^{jkz-j\omega t}$ and $\vec{H} = (k/\omega\mu)(a_{\hat{Y}} + j\hat{a}_X)e^{jkz-j\omega t}$, the time-averaged pointing vector is

(a) Null vector
(b) $(k/\omega\, u)\hat{a}_z$
(c) $(2k/\omega\, u)\hat{a}_z$
(d) $(k/2\,\omega\, u)\hat{a}_z$

[GATE 2004: 2 Marks]

Soln. $\boldsymbol{\vec{E} = (\hat{a}_x + j\,\hat{a}_y)\, e^{jkz-j\omega t}\ \ v/m}$

Is travelling in the positive z – direction

$$\vec{H} = \left(\frac{K}{\omega_\mu}\right)(a_{\hat{y}} + j a_x)\ e^{jkz-j\omega t}\ \ v/m$$

Is travelling in the positive z – direction

The time averaged pointing vector

$$\vec{P} = \frac{1}{2}\vec{E} \times \vec{H}$$

$$= \frac{1}{2}\left[(\hat{a}_x + j\hat{a}_y)\ \times \frac{K}{\omega_u}\,(\hat{a}_y + j\hat{a}_x)\right]$$

$$= \frac{1}{2}\left[\frac{K}{\omega_\mu}\hat{a}_z - \frac{K}{\omega_\mu}\hat{a}_z\right] = 0$$

$$\vec{P} = 0$$

Option (a)

14. The value of the integral of the function $g(x,y) = 4x^3 + 10\,y^4$ along the straight line segment from the point (0,0) to the point (1,2) in the x-y plane is

(a) 33
(b) 35
(c) 40
(d) 56

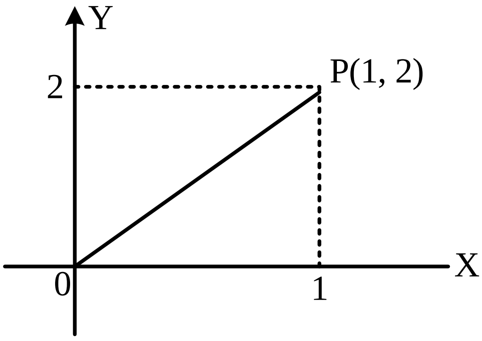
[GATE 2008: 2 Marks]

Soln. $\boldsymbol{g(x,y) = 4x^3 + 10y^4}$

$$I = \int g(x,y)$$

Along the straight line segment OP as shown in figure

Any point on OP satisfies y = 2x



$$\boldsymbol{g(x,y) = 4x^3 + 10(2x)^4}$$

$$\boldsymbol{= 4x^3 + 160\,x^4}$$

$$\boldsymbol{I = \int_0^1 (4x^3 + 160\,x^4)dx}$$

$$\boldsymbol{= \left[4\frac{x^4}{4}\right]_0^1 + \left[160\frac{x^5}{5}\right]_0^1}$$

$$\boldsymbol{= (1-0) + 32(1-0)}$$

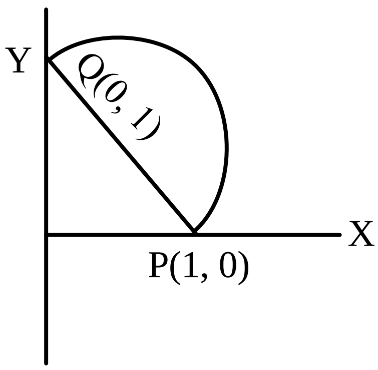
$$\boldsymbol{= 33}$$

Option (a)

15. Consider points P and Q in the x-y plane, with P = (1, 0) and Q = (0, 1). The line integral $2\int_P^Q (x\,dx + y\,dy)$ along the semicircle with the line segment PQ as its diameter
(a) is – 1
(b) is 0
(c) is 1
(d) depends on the direction (clockwise or anti-clockwise) of the semicircle

[GATE 2008: 2 Marks]

Soln. $\boldsymbol{P(1,0)}$ ***and*** $\boldsymbol{Q(0,1)}$ **are points in X – Y plane**



$$\boldsymbol{I = 2\int_{P}^{Q} x\,dx + y\,dy}$$

$$= x^2 + y^2 \Big|_{P}^{Q}$$

$$= x^2\Big|_{1}^{0} + y^2\Big|_{0}^{1}$$

$$\boldsymbol{= -1 + 1 = 0}$$

Option (b)

16. If a vector field $\vec{V}$ is related to another vector field $\vec{A}$ through $\vec{V} = \nabla \times \vec{A}$, which of the following is true? Note: C and S_C refer to any closed contour and any surface whose boundary is C.

(a) $\oint_C \vec{V}\,.d\vec{l} = \iint_{SC} \vec{A}\,.d\,\vec{s}$

(b) $\oint_C \vec{A}\,.d\vec{l} = \iint_{SC} \vec{V}\ .\overrightarrow{ds}$

(c) $\oint_C \nabla \times \vec{V}\,.d\vec{l} = \iint_{SC} (\nabla \times \bar{A})\ .\overrightarrow{ds}$

(d) $\oint_C \nabla \times \vec{A} \,.\, d\vec{l} = \iint_{sc} \vec{A} \,.\, \overrightarrow{ds}$

[GATE 2009: 2 Marks]

Soln. $\vec{V} = \nabla \times \vec{A} \qquad - - - - - - -(I)$

$$\iint \vec{V}.\overrightarrow{ds} = \iint (\nabla \times \vec{A})\, \overrightarrow{ds}$$

According to Stroke's theorem

$$\iint \vec{V} \,.\overrightarrow{ds} = \oint \vec{A} \,.\overrightarrow{dl}$$

Option (b)

17. If $\vec{A} = xy\hat{a}_x + x^2\hat{a}_y, then \oint_C \vec{A} \,.\, \overline{dl}$ over the path shown in the Figure is

(a) 0

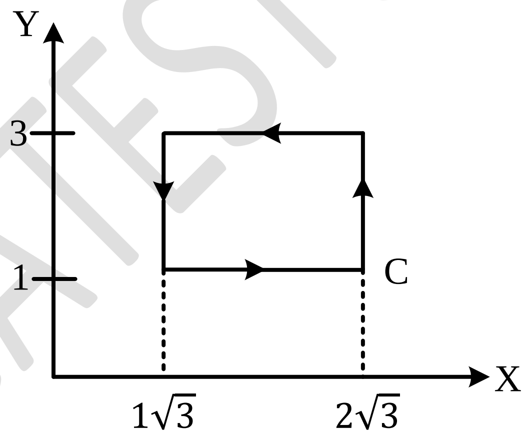
(b) $\frac{2}{\sqrt{3}}$

(c) 1

(d) $2\sqrt{3}$

[GATE 2010: 2 Marks]

Soln.



If $\vec{A} = xy\hat{a}_x + x^2\hat{a}_y$

$$\oint \vec{A}\,.\overrightarrow{dl} = I$$

Is evaluated over the path as shown below

$$\boldsymbol{I = \oint \vec{A}\ .\overrightarrow{dl} = \oint \vec{A}\ .\left(dx\, a_x + dy\, a_y\right)}$$

$$I = \oint \vec{A}\ .\, d_x\ \vec{a}_x\ , Y = 1, x\ varies\ from\ \frac{1}{\sqrt{3}}\ to\ \frac{2}{\sqrt{3}}$$

$$\boldsymbol{+ \int \bar{A}\ .\, dy\, \vec{a}_y\ , x = \frac{2}{\sqrt{3}}\ , y\ varies\ from\ 1\ to\ 3}$$

$$\boldsymbol{+ \int \vec{A}\ .\, dy\, \vec{a}_x\ , y = 3\ , x\ varies\ from\ \frac{2}{\sqrt{3}}\ to\ \frac{1}{\sqrt{3}}}$$

$$\boldsymbol{+ \int \bar{A}\ .\, dy\, \vec{a}_y\ , x = \frac{1}{\sqrt{3}}\ , y\ varies\ from\ 3\ to\ 1}$$

$$\boldsymbol{I = \oint \vec{A}\ .\overrightarrow{dl} = \int_{1/\sqrt{3}}^{2\sqrt{3}} xy\ d_x + \int_{1}^{3} x^2\ d_y + \int_{2/\sqrt{3}}^{1/\sqrt{3}} xy\ d_x + \int_{3}^{1} x^2\ d_y}$$

$$= y\ \frac{x^2}{2}\Bigg|_{1\sqrt{3}}^{2\sqrt{3}} + x^2\ y\Bigg|_{1}^{3} + y\ \frac{x^2}{2}\Bigg|_{2\sqrt{3}}^{1\sqrt{3}} + x^2\ y\Bigg|_{3}^{1}$$

$$\boldsymbol{= \frac{1}{2}\left(\frac{4}{3} - \frac{1}{3}\right) + \frac{4}{3}(3-1) + \frac{3}{2}\left(\frac{1}{3} - \frac{4}{3}\right) + \frac{1}{3}(1-3)}$$

$$\boldsymbol{= \frac{1}{2} + \frac{4}{3} \times \frac{2}{1} + \frac{3}{2}(-1) + \frac{1}{3}(-2)}$$

$$\boldsymbol{= \frac{1}{2} + \frac{8}{3} - \frac{3}{2} - \frac{2}{3}}$$

$$= \frac{19}{6} - \frac{3}{2} - \frac{2}{3} = 1$$

Option (c)

18. A current sheet $\vec{J} = 10\hat{u}_x$ A/m lies on the dielectric interface x = 0 between two dielectric media with $\varepsilon_{r1} = 5, \mu_{r2} = 1$ in Region $1 (x < 0)$ and $\varepsilon_{r2} = 2, \mu_{r2} = 2$ in Region 2 $(x > 0)$. If the magnetic field in Region 1 at $x = 0^-$ is $\vec{H}_1 = 3\,\hat{u}_x + 30\,\hat{u}_y$ A/m, the magnetic field in Region $-2\ at\ x = 0^+$ is

(a) $\vec{H}_2 = 30\,\hat{u}_x + 30\,\hat{u}_y - 10\,\hat{u}_z$ A/m

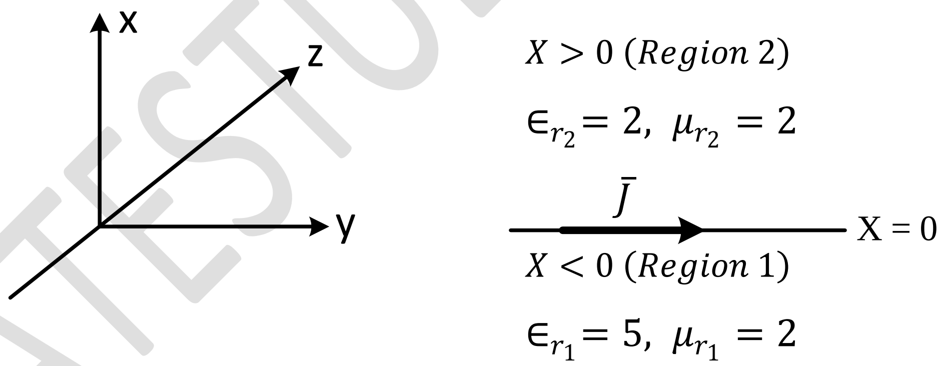
(b) $\vec{H}_2 = 3\,\hat{u}_x + 30\,\hat{u}_y - 10\,\hat{u}_z$ A/m

(c) $\vec{H}_2 = 1.5\,\hat{u}_x + 40\,\hat{u}_y$ A/m

(d) $\vec{H}_2 = 3\,\hat{u}_x + 30\,\hat{u}_y + 10\,\hat{u}_z$ A/m

[GATE 2011: 2 Marks]

Soln.



The dielectric interface X = 0, y – z plane current sheet $\bar{J} = 10\hat{u}_y\ A/m$

$$J = \hat{n} \times H$$

Where $\hat{n}$ is the unit vector along the outward normal to the surface.

The gives rise to a H filed of 10 A/m at X = 0^+ in the negative Z direction or $H_{Z1} = -10$

$$\bar{H}_1 = 3\hat{u}_x + 30\hat{u}_y\ A/m\ at\ X = \bar{0}$$

$H_{X1} = 30$ in y direction is tangential component

$H_{X1} = 3$ **in the X – direction is normal component.**

The normal component of $\bar{B}$ is continuous across the boundary

$$B_{n1} = B_{n2}$$

$$\mu_1 H_{x_1} = \mu_2 H_{x_2}$$

$$H_{x_2} = \frac{\mu_1}{\mu_2} . H_{x_1}$$

$$= \frac{1}{2} \times 3 = 1.5$$

Magnetic field in the region 2 at $X = 0^+$ is

$$\bar{H}_2 = H_{x_2} \hat{u}_x + H_{y_2} \hat{u}_y + H_{z_2} \hat{u}_z \ A/m$$

$$= 1.5\hat{u}_x + 30\hat{u}_y - 10\hat{u}_z \ A/m$$

Option (a)