Basics of Electromagnetics – Maxwell's Equations (Part - I)

1. $\oint_C \vec{A}\,.\,\vec{dl} = \oint_{C}\_\_.\,\vec{d}\,S$

[GATE 1994: 1 Mark]

Soln.

$$\oint \overline{\boldsymbol{A}}\ .\,\overline{\boldsymbol{dl}} = \iint \nabla \times \vec{\boldsymbol{A}}\ .\,\overrightarrow{\boldsymbol{da}}\ \ \textbf{using Stoke's Theorem}$$

$$= \oint_{S} \nabla \times \vec{\boldsymbol{A}}\,.\,\overline{\boldsymbol{ds}}$$

2. The electric field strength at distant point, P, due to a point charge, +q, located at the origin, is 100 µ V/m. If the point charge is now enclosed by a perfectly conducting metal sheet sphere whose center is at the origin, then the electric field strength at the point, P, outside the sphere, becomes
   (a) Zero
   (b) 100 µV/m
   (c) – 100 µV/m
   (d) 50 µV/m

[GATE 1995 : 1 Mark]

Soln. The point charge +q will induce a charge – q on the surface of metal sheet sphere. Using Gauss's law, the net electric fulx passing through a closed surface is equal to the charge enclosed = + q – q = 0

D = 0, E = 0 at point P.

Option (a)

3. A metal sphere with 1 m radius and surface charge density of 10 Coulombs / m^2 is enclosed in a cube of 10 m side. The total outward electric displacement normal to the surface of the cube is
   (a) 40 π Coulombs
   (b) 10 π Coulombs
   (c) 5 π Coulombs
   (d) None of the above

[GATE 1996: 1 Mark]

Soln. The sphere is enclosed in a cube of side = 10m. using Gauss's law, the net electric flux flowing out through a closed surface is equal to charge enclosed.

$$\oiint_S \vec{D} \,.\, \overrightarrow{da} = Q(enclosed)$$

$$= P_S \, 4\pi r^2$$

$$= 10 \times 4\pi$$

$$= 40\pi \; coulombs$$

Option (a)

4. The Maxwell's equation, $\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ is based on

(a) Ampere's law
(b) Gauss's law
(c) Faraday's law
(d) Coulomb's law

[GATE 1998: 1 Mark]

Soln. Ampere's law states that the magneto motive force around a closed path is equal to the current enclosed by the path

For steady electric fields

$$\oint_C \vec{H} \,.\, \overrightarrow{dl} = I = \iint \vec{J} \,.\, \overrightarrow{da}$$

$\vec{J} = \overline{\sigma E}$ **is the conduction**

Current density (amp/m^2)

For time – varying electric fields:

$$\oint \vec{H} \,.\, \overrightarrow{dl} = \iint \left(\vec{J} + \overrightarrow{J_d}\right) \,.\, \overrightarrow{d_a}$$

Where $\overrightarrow{J_d}$ is the displacement current density $\frac{\partial \vec{D}}{\partial t}$

By Stroke's theorem

$$\oint \vec{H} \,.\, \overrightarrow{dl} = \iint \left(\nabla \times \vec{H}\right) \,.\, \overrightarrow{da}$$

So $\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

$\vec{J} + \frac{\partial \vec{D}}{\partial t}$ **is the total current density (Conduction current density + displacement current density)**

Option (a)

5. The time averaged Poynting vector, in W/m^2, for a wave with $\vec{E} = 24e^{j(\omega t+\beta z)}\overrightarrow{a_y}$ V/m in free space is

(a) $-\frac{2.4}{\pi}\overrightarrow{a_z}$

(b) $\frac{2.4}{\pi}\overrightarrow{a_z}$

(c) $\frac{4.8}{\pi}\overrightarrow{a_z}$

(d) $-\frac{4.8}{\pi}\overrightarrow{a_z}$

[GATE 1998: 1 Mark]

Soln. $E = 24e^{j(\omega t+\beta Z)}\overrightarrow{ax}$

The wave is travelling in negative Z direction with $|E| = 24\ V/m$.

Poynting vector $\vec{P} = \vec{E} \times \vec{H}$ $\quad$ ***instant areous power/m^2***

$$\frac{E}{H} = \eta_0 = 120\pi$$

Time averaged Poynting vector

$$\overrightarrow{P_{avg}} = P_Z(-\vec{a}_Z)$$

$$= \frac{|E|^2}{2\eta_0} = \frac{(24)^2}{2\times 120\pi}$$

$$= \frac{2.4}{\pi}\ W/m^2$$

$$\overrightarrow{P_{avg}} = \frac{-2.4}{\pi}\vec{a}_Z\ W/m^2$$

Option (a)

6. A loop is rotating about the y – axis in a magnetic field $\vec{B} = B_0\cos(\omega t+\phi)\ \overrightarrow{a_x}\ T$. The voltage in the loop is

(a) zero

(b) due to rotation only

(c) due to transformer action only

(d) due to both rotation and transformer action

[GATE 1998: 1 Mark]

Soln. **The magnetic field changing with time**

$$\overline{B} = B_0\cos(\omega t+\phi)\,\overline{ax}\ T$$

The voltage induced in a stationary loop due to time changing magnetic filed as given by Faraday's law is

$$V_i = -\iint \frac{\partial \vec{B}}{\partial t} \cdot \overrightarrow{da}$$

The voltage induced in a loop moving with velocity v in steady magnetic field is

$$V_m = \oint_C (\vec{v} \times \vec{B}) = -\iint \frac{\partial \vec{B}}{\partial t} \cdot \overrightarrow{da} + \oint_C (\vec{v} \times \vec{B}) \cdot \overrightarrow{dl}$$

The voltage induced in loop is due to both rotation and transformer action.

Option (d)

7. An electric field on a plane is described by its potential $V = 20(r^{-1} + r^{-2})$ where r is the distance from the source. The field is due to
(a) a monopole
(b) a dipole
(c) both a monopole and a dipole
(d) a quadrupole

[GATE 1999: 1 Mark]

Soln. The potential V_1 at a point P due to a monopole Q is given by

$$V_1 = \frac{Q}{4\pi \in_o r}$$

The potential V_2 at the distant point P due to it's dipole is

$$V_2 = \frac{Q\, d \cos\theta}{4\pi \in_0 r}$$

$$V = V_1 + V_2 = \frac{K}{r} + \frac{K}{r^2}$$

Option (c)

8. Identify which one of the following will NOT satisfy the wave equation.
(a) $50^{e^{j(\omega t - 3z)}}$
(b) $\sin[\omega(10z + 5t)]$
(c) $\cos(y^2 + 5t)$
(d) $\sin(x) \cdot \cos(t)$

[GATE 1999: 1 Mark]

Soln. $f(x - ct)$ *or* $f(\omega t - \beta x)$ $----------$

Represents wave travelling in the positive x direction with velocity $V = C \quad or \quad V = \frac{\omega}{\beta}$

Accordingly $50e^{j(\omega t - \beta z)}$ represents wave travelling in positive Z direction with velocity $V = \frac{\omega}{\beta}$

$\sin \omega\,(10z + 5t) = \sin 5(\omega t + \beta z)\,, \beta = 2\omega$

Represents wave travelling in the negative Z direction

$$\sin(x)\cos(t) = \frac{1}{2}[\sin(x+t) + \sin(x-t)]$$

Represents standing wave consisting of two waves: one travelling in negative 'x' direction and other travelling in positive 'x' direction.

$\cos(y^2 + 5t)$ is not of the form $f(x - ct)$ or $f(\omega t - \beta x)$. This function does not satisfy the wave equation.

Option (c)

9. The unit of $\nabla \times H$ is
   (a) Ampere
   (b) Ampere/meter
   (c) Ampere/meter^2
   (d) Ampere-meter

[GATE 2003: 1 Mark]

Soln. $\nabla \times H = J + \frac{\partial D}{\partial t} \;\; amps/m^2$

Option (c)

10. $\nabla \times \nabla \times P$, where P is a vector, is equal to
   (a) $P \times \nabla \times P - \nabla^2 P$
   (b) $\nabla^2 P + \nabla(\nabla \cdot P)$
   (c) $\nabla^2 P + \nabla \times P$
   (d) $\nabla(\nabla \cdot P) - \nabla^2 P$

[GATE 2006: 1 Mark]

Soln. $\nabla \times \nabla \times P = \nabla(\nabla . P) - \nabla^2 P$

Option (d)

11. $\iint (\nabla \times P)$. Ds, Where P is a vector, is equal to
   (a) $\oint P \,.\, dl$
   (b) $\oint \nabla \times \nabla \times P \,.\, dl$

(c) $\oint \nabla \times P \,.\, dl$ (d) $\iiint \nabla \,.\, P \; dv$

[GATE 2006: 1 Mark]

Soln.

$$\iint (\nabla \times \boldsymbol{P}) \;.\overrightarrow{\boldsymbol{ds}} = \oint_L \vec{\boldsymbol{P}} \,.\overrightarrow{\boldsymbol{dl}} \quad \textbf{using Stroke's Theorem}$$

Option (a)

12. If C is a closed curve enclosing a surface S, then the magnetic field intensity $\vec{H}$, the current density $\vec{J}$ and the electric flux density $\vec{D}$ are related by

(a) $\iint_S \vec{H}.d\vec{s} = \int_C \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right).d\vec{l}$

(b) $\iint_S \vec{H}.d\vec{l} = \int_C \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right).d\vec{s}$

(c) $\iint_S \vec{H}.d\vec{s} = \int_C \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right).d\vec{l}$

(d) $\iint_C \vec{H}.d\vec{l} = \int_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right).d\vec{s}$

[GATE 2007: 1 Mark]

Soln. For time varying fields, the ampere's law relating $\vec{H}, \vec{J}$ and $\bar{D}$ is given as:

$$\nabla \times \vec{\boldsymbol{H}} = \vec{\boldsymbol{J}} + \frac{\partial \vec{\boldsymbol{D}}}{\partial t}$$

By stroke's theorem

$$\oint \vec{\boldsymbol{H}} \;.\overrightarrow{\boldsymbol{dl}} = \iint_S (\nabla \times \vec{\boldsymbol{H}}) \,.\overrightarrow{\boldsymbol{ds}} = \iint_S \left(\vec{\boldsymbol{J}} + \frac{\partial \vec{\boldsymbol{D}}}{\partial t}\right) \;.\overrightarrow{\boldsymbol{ds}}$$

Option (d)

13. For static electric and magnetic fields in an in homogenous source-free medium, which of the following represents the correct form of two of Maxwell's equations?

(a) $\nabla . E = 0$
$\nabla \times B = 0$

(b) $\nabla . E = 0$
$\nabla . B = 0$

(c) $\nabla \times E = 0$
$\nabla \times B = 0$

(d) $\nabla \times E = 0$
$\nabla . B = 0$

[GATE 2008: 1 Mark]

Soln. Considering the Maxwell's equations for electromagnetic fields,

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \times E = -\frac{\partial \vec{D}}{\partial t}$$

$$\nabla . \vec{D} = \rho$$

$$\nabla . \vec{B} = 0$$

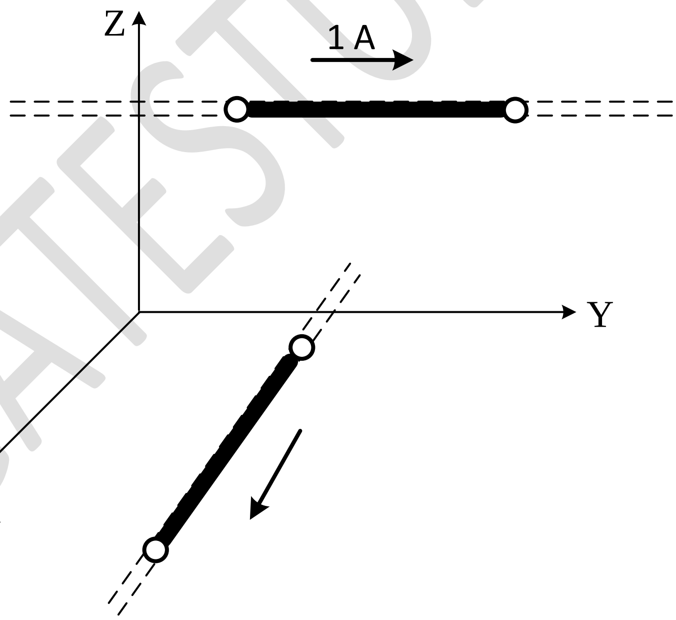
For static electric and magnetic fields

$$\nabla \times \vec{H} = \vec{J} \qquad \nabla . \vec{D} = \rho$$

$$\nabla \times \vec{E} = 0 \qquad \nabla . \vec{B} = 0$$

Option (d)

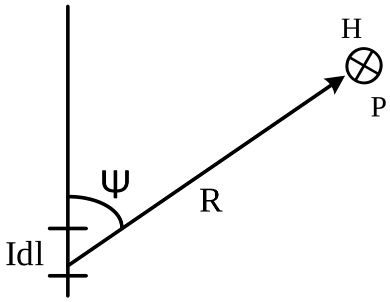
14. Two infinitely long wires carrying current are as shown in the Fig below. One wire is in the y-z plane and parallel to the y-axis. The other wire is in the x-y plane and parallel to the x-axis. Which components of the resulting magnetic field are non-zero at the origin?



(a) x, y, z components
(b) x, y components
(c) y, z components
(d) x, z components

[GATE 2009: 1 Mark]

Soln. When a current I flows in a closed circuits, the magnetic field strength H at any point is a result of this current flow



$$dH = \frac{Idl \times \hat{R}}{4\pi R^2}$$

$\hat{R}$ is the unit vector in direction of R. The direction of H is perpendicular to the plane containing Idl and R.in the direction in which right hand screw would move in turning from Idl to R. The first current element is in Y direction and $\hat{R}$ is in Z direction. H is in X direction.

The second current element is in X direction and $\hat{R}$ is in Y direction. H is in Z direction

Resultant $B = \overrightarrow{B_1} + \overrightarrow{B_2}$

$$B_0(K_1\overrightarrow{a_x} + K_2\overrightarrow{a_z})$$

Option (d)

15. The electric field component of a time harmonic plane EM wave traveling in a non-magnetic lossless dielectric medium has amplitude of 1 V/m. If the relative permittivity of the medium is 4, the magnitude of the time-average power density vector (in W/m^2) is

(a) $\frac{1}{30\pi}$ (c) $\frac{1}{120\pi}$

(b) $\frac{1}{60\pi}$ (d) $\frac{1}{240\pi}$

[GATE 2010: 1 Mark]

Soln. Time average power density $= \frac{1}{2}EH$

$$P_{av} = \frac{1}{2} \times \frac{E^2}{\eta}$$

Intrinsic impedance of EM wave $\gamma = \sqrt{\frac{\mu}{\epsilon}}$

$$\eta = \sqrt{\frac{\mu_0}{4\epsilon_0}} = \frac{120\pi}{2} = 60\pi$$

$$P_{av} = \frac{E^2}{2\eta} = \frac{1}{2\times 60\pi}$$

$$= \frac{1}{120\pi}$$

Option (c)

16. Consider a closed surface S surrounding a volume V. If $\hat{r}$ is the position vector of a point inside S, with $\hat{n}$ the unit normal on S, the value of the integral $\oint 5\vec{r}\,.\,\hat{n}\; ds$ is

(a) 3 V
(b) 5 V
(c) 10 V
(d) 15 V

[GATE 2011: 1 Mark]

Soln. **S is a closed surface surrounding a volume V**

$$I = \oiint 5\bar{r}.\vec{n}\; ds$$

In spherical coordinates, differential area in $\overrightarrow{a_r}$ direction

in $\vec{a}_r$ direction

$$ds = (r d\theta)\,.\,(r \sin\theta\; d\phi) = r^2 \sin\theta\; d\theta\; d\phi$$

$$I = \iint 5r\;.\vec{n}\, r^2 \sin\theta\;\; d\theta\; d\phi$$

$$= \int_0^{\pi}\int_0^{2\pi} 5\, r^3 \sin\theta\;\; d\theta\; d\phi$$

$$= 5r^3 \int_0^{\pi} \sin\theta\; d\theta \int_0^{2\pi} d\phi$$

$$= 5r^3 (2)(2\pi) = 20\pi r^3$$

$$= 15\left(\frac{4}{3}\pi r^3\right) \quad volume\ of\ sphere = \frac{4}{3}\pi r^3$$

$$= 15V$$

Option (d)

17. Consider the following statements regarding the complex Poynting vector $\hat{P}$ for the power radiated by a point source in an infinite homogeneous and lossless medium. $Re(\hat{P})$ denotes the real part of $\hat{P}$, S denotes a spherical surface whose centre is at the point source, and $\hat{n}$ denotes the unit surface normal on S. Which of the following statements is **TRUE?**

(a) $Re(\hat{P})$ remains constant at any radial distance from the source

(b) $Re(\hat{P})$ increases with increasing radial distance from the source

(c) $\oint_S Re\,(\vec{P}).\hat{n}$ d S remains constant at any radial distance from the source

(d) $\oint_S Re(\vec{P}).\hat{n}$ d S decreases with increasing radial distance from the source.

Soln. $\oint_S \boldsymbol{R_e}(\overline{\boldsymbol{P}}).\hat{\boldsymbol{n}}\,\boldsymbol{ds}$ **gives average power and it decreases with increasing radial distance from the source**

$\overline{\boldsymbol{P}} = \overline{\boldsymbol{E}} \times \overline{\boldsymbol{H}}$ **is a measure of energy flow per unit area.** $\boldsymbol{watts/m^2}$

Option (d)

18. Consider a vector field $\bar{A}(\bar{r})$. The closed loop line integral $\oint \vec{A}\,.\overrightarrow{dl}$ can be expressed as

(a) $\oiint(\nabla \times \vec{A})\,.d\vec{s}$ over the closed the closed volume bounded by the loop

(b) $\oiiint(\nabla\,.\,\vec{A})$ dv over the closed volume bounded by the loop

(c) $\iiint(\nabla \times \vec{A})$ dv over the open volume bounded by the loop

(d) $\iint(\nabla \times \vec{A})\,.d\vec{s}$ over the open surface bounded by the loop

[GATE 2013: 1 Mark]

Soln. $\oint \overrightarrow{\boldsymbol{A}}\,.\overrightarrow{\boldsymbol{dl}} = \iint(\nabla \times \overline{\boldsymbol{A}})\,.\overrightarrow{\boldsymbol{ds}}$ **over the open surface bounded by the loop, using Stroke's theorem.**

Option (d)

19. The divergence of the vector field $\vec{A} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$ is

(a) 0	(c) 1
(b) 1/3	(d) 3

[GATE 2013: 1 Mark]

Soln. $\vec{A} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$

$$\boldsymbol{dia\vec{A} = \nabla . \vec{A}}$$

$$= \left(\frac{\partial i}{\partial x} + \frac{\partial i}{\partial y} + \frac{\partial k}{\partial z}\right).\left(x\hat{a}_x + y\hat{a}_y + z\hat{a}_z\right)$$

$$= \left(\frac{\partial i}{\partial x} + \frac{\partial i}{\partial y} + \frac{\partial k}{\partial z}\right).(\boldsymbol{xi + yi + 2k})$$

= 1 + 1 + 1 = 3

$$\boldsymbol{i\,.\,i = i.i = k\,.\,k = 1}$$

Option (d)

20. The force on a point charge +q kept at a distance d from the surface of an infinite grounded metal plate in a medium of permittivity ϵ is

(a) 0

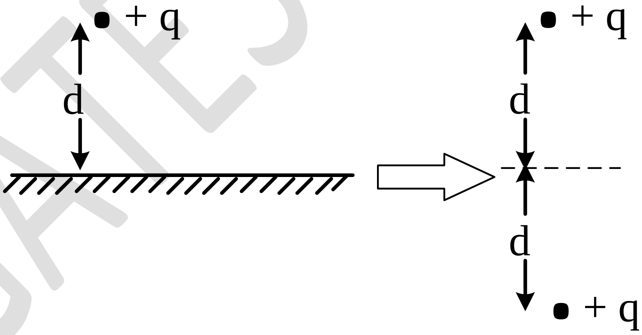
(b) $\frac{q^2}{16\pi \in d^2}$ away from the plate

(c) $\frac{q^2}{16\pi \in d}$ towards the plate

(d) $\frac{q^2}{4\pi \in d^2}$ towards the plate

[GATE 2014: 1 Mark]

Soln.



$$\boldsymbol{Force\ \ \bar{F} = \frac{q \times q}{4\pi\varepsilon(2d)^2}}$$

$$= \boldsymbol{\frac{q^2}{16\pi\epsilon d^2}}$$

Force is attractive and towards the plate

Option (c)