Plane Waves – GATE Problems (Part – I)

1. A plane electromagnetic wave traveling along the + z – direction, has its electric field given by $E_x = 2\cos(\omega t)$ *and* $E_y = 2\cos(\omega + 90^0)$ the wave is
   (a) linearly polarized
   (b) right circularly polarized
   (c) left circularly polarized
   (d) elliptically polarized

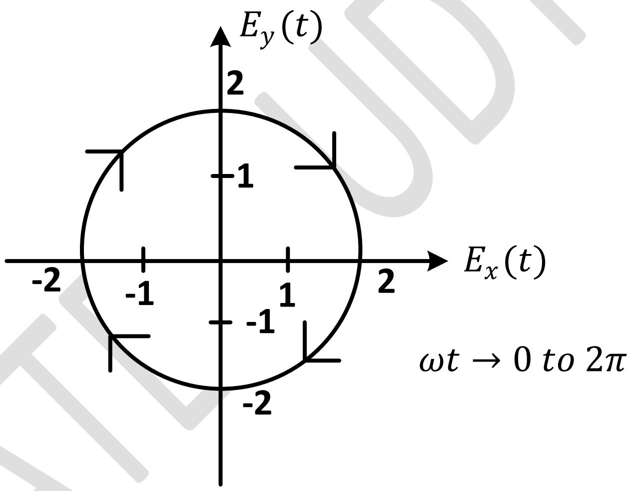
[GATE 1994: 1 Mark]

Soln. $\boldsymbol{E_x(t) = 2\cos\omega t}$

$\boldsymbol{E_y(t) = 2\cos(\omega t + 90^0) = -2\sin\omega t}$

$\boldsymbol{E_x^2(t) + E_y^2(t) = 2^2}$

It represents a circle in the $\boldsymbol{E_x - E_y}$ plane with radius 2 as shown in figure. Hence the wave is circularly polarized.



$\boldsymbol{\omega t = 0}$	$\boldsymbol{\pi/2}$	$\boldsymbol{\pi}$	$\boldsymbol{\frac{3\pi}{2}}$	$\boldsymbol{2\pi}$
$\boldsymbol{E_x = 2}$	**0**	**− 2**	**0**	**2**
$\boldsymbol{E_y = 0}$	**− 2**	**0**	**2**	**0**

When the fingers of the left hand follows the clock wise direction (direction of rotation of the E vector), the thumb is pointing in the given direction of propagation (+ z direction). The wave is left circularly polarized.

Option (c)

2. The intrinsic impedance of a lossy dielectric medium is given by

(a) $\frac{j\omega\mu}{\sigma}$

(b) $\frac{j\omega\in}{\mu}$

(c) $\sqrt{\frac{j\omega\mu}{(\sigma+j\omega\epsilon)}}$

(d) $\sqrt{\frac{\mu}{\in}}$

[GATE 1995: 1 Mark]

Soln. **Conductivity =** $\boldsymbol{\sigma\ mhos/m}$

Permittivity = $\boldsymbol{\in farad\ /\ m}$

Permeability = $\boldsymbol{\mu\ henry\ /\ m}$

E / H for a lossy dielectric medium $\boldsymbol{= \eta = \sqrt{\frac{j\omega\mu}{\sigma+j\omega\epsilon}}}$

Option (c)

3. Copper behaves as a

(a) Conductor always.

(b) Conductor or dielectric depending on the applied electric field strength

(c) Conductor or dielectric depending on the frequency

(d) Conductor or dielectric depending on the electric current density

[GATE 1995: 1 Mark]

Soln. **For a conductor** $\boldsymbol{\sigma \gg \omega\epsilon}$

For copper with $\boldsymbol{\sigma = 5.8 \times 10^7\ mhos/m}$

$$\boldsymbol{\in = \in_0 = \frac{1}{36\pi\times10^9}\ farad/m}$$

at relatively large frequency

$$\boldsymbol{f = 3 \times 10^{15}\ Hz}$$

$$\boldsymbol{\frac{\sigma}{\omega\epsilon} = \frac{5.8 \times 10^7 \times 36\pi \times 10^9}{2\pi \times 3 \times 10^{15}} = 348}$$

Copper is good conductor for the frequencies used in practice

Option (a)

4. The intrinsic impedance of copper at high frequency is
   (a) Purely resistive
   (b) Purely inductive
   (c) Complex with a capacitive component
   (d) Complex with a inductive component

[GATE 1998: 1 Mark]

Soln. The intrinsic impedance $\boldsymbol{\eta} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$

For a good conductor $\boldsymbol{\sigma \gg \omega} \in$

$$\boldsymbol{\eta} = \sqrt{\frac{j\omega\mu}{\sigma}}$$

$$= \sqrt{\frac{\omega\mu}{\sigma}} \quad e^{j45^0}$$

$$= \boldsymbol{\eta_R + j\eta_x}$$

$$\boldsymbol{\eta_R = \eta_x} = \sqrt{\frac{\omega\mu}{2\sigma}}$$

η is complex with inductive component

Option (d)

5. The wavelength of wave with propagation constant $(0.1\,\pi + j\,0.2\pi)m^{-1}$ is
   (a) $\frac{2}{\sqrt{0.05}}m$
   (b) 10 m
   (c) 20 m
   (d) 30 m

[GATE 1998: 1 Mark]

Soln. Propagation constant $= \boldsymbol{\alpha + j\beta}$

$$= (\mathbf{0.1}\boldsymbol{\pi} + \boldsymbol{j}\ \mathbf{0.2}\boldsymbol{\pi})\boldsymbol{m}^{-\mathbf{1}}$$

$$\boldsymbol{\alpha} \;=\; \textbf{attenuation constant} \;=\; \mathbf{0.1\ \pi}$$

$$\boldsymbol{\beta} \;=\; \textbf{phase constant} \;=\; \mathbf{0.2\ \pi}$$

wavelength $\boldsymbol{\lambda} = \frac{2\pi}{\beta} = \frac{2\pi}{0.2\pi} = \mathbf{10}\boldsymbol{m}$

Option (b)

6. The depth of penetration of a wave in a lossy dielectric increases with increasing
   (a) Conductivity
   (b) Permeability
   (c) Wavelength
   (d) Permittivity

[GATE 1998: 1 Mark]

Soln. Depth of penetration $\delta = \frac{1}{\alpha}$

α = **attenuation constant**

γ = **Propagation constant**

$= \alpha + j\beta$

$= \sqrt{j\omega\mu\,(\sigma + j\omega\epsilon)}$

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}} - 1\right]}$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}} + 1\right]}$$

For a lossy dielectric with $\sigma \neq 0$

α increases with increasing μ and σ good conductor with $\frac{\sigma}{\omega\epsilon} \gg 1$

$$\alpha = \beta = \omega\sqrt{\frac{\mu\epsilon}{2}\,\frac{\sigma}{\omega\epsilon}}$$

$$= \sqrt{\frac{\omega^2\mu\sigma}{2\omega}}$$

$$= \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$\alpha \propto f \quad or \quad \delta \propto \frac{1}{f} \quad or\ \delta \propto \lambda$$

Option (c)

7. The polarization of a wave with electric field vector
$\vec{E} = E_0 e^{j(\omega t - \beta z)}(\vec{a}_x + \vec{a}_y)$

(a) Linear
(b) Elliptical
(c) Left hand circular
(d) Right hand circular

[GATE 1998: 1 Mark]

Soln. $\vec{E} = E_0 e^{j(\omega t - \beta z)}(\vec{a}_x + \vec{a}_y)$

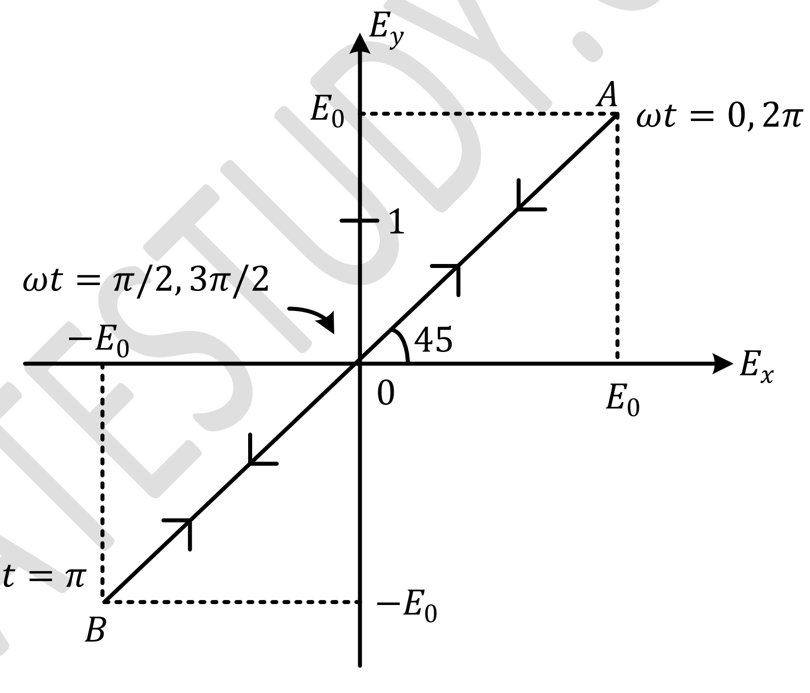
It is a wave propagating in Z direction with electric field components in x and y direction

at $z = 0$

$$E_x = E_0 \cos \omega t$$

$$E_y = E_0 \cos \omega t$$

$E_y = E_x$ **at any time t**



As ωt varies from 0 to π the tip of $\vec{E}$ vector moves along the straight line AB from A to B and as ωt varices from π to 2π the tip of $\vec{E}$ vector moves back from B to A and the cycle repeats. The polarization of the wave is linear

Option (a)

8. A TEAM wave is incident normally upon a perfect conductor. The E and H fields at the boundary will be respectively

(a) Minimum and minimum
(b) Maximum and maximum
(c) Minimum and maximum
(d) Maximum and minimum

[GATE 2000: 1 Mark]

Soln. IN the case of a plane wave incident normally upon the surface of a perfect conductor, the wave is entirely reflected, neither E nor H can exist within a perfect conductor.

According to the boundary condition

$$E_{tangential} = E_{inc} + E_{ref}$$
$$= 0$$

E_{inc} is reflected with phase reversal

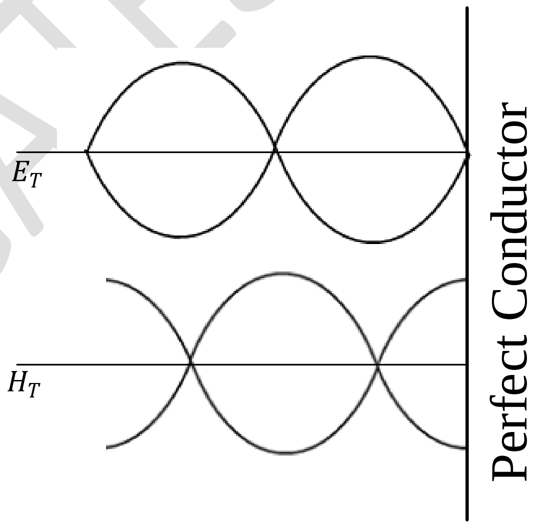
E is minimum equal to zero at the surface of a perfect conductor

The magnetic field H must be reflected without reversal of phase. It both E and H are reversed with phase reversal there would be no reversal of direction of propagation. The phase of the reflected magnetic field strength H_r is the same.

$$H_{tan} = H_i + H_r = 2H_i = J_S$$

Where $\vec{J}_S$ is the linear current density in A/m on the surface.

H is maximum equal to twice the incident value at the boundary



Option (c)

9. If a plane electromagnetic wave satisfies the equation $\frac{\partial^2 E_x}{\partial z^2} = c^2 \frac{\partial^2 E_x}{\partial t^2}$, the wave propagates in the

(a) x – direction
(b) z – direction
(c) y – direction
(d) x z plane at an angle of 45^0 between the x and z directions

[GATE 2001: 1 Mark]

Soln. The wave equations for free space (In a perfect dielectric containing no charges)

$$\nabla^2 E = \mu\epsilon \ddot{E}$$

$$\nabla^2 H = \mu\epsilon \ddot{H}$$

The wave equation reduces to a simple form where E and H are considered to be independent of two dimensions (x and y)

$$\frac{\partial^2 E}{\partial z^2} = \mu\epsilon \frac{\partial^2 E}{\partial t^2}$$

For uniform plane propagating in the Z direction, E may have components E_x and E_y

$$\frac{\partial^2 E_x}{\partial z^2} = \mu\epsilon \frac{\partial^2 E_x}{\partial t^2}$$

Where $v_0 = \frac{1}{\sqrt{\mu\epsilon}}$ is the velocity of propagation

Option (b)

10. The depth of penetration of electromagnetic wave in a medium having conductivity σ at a frequency 1 KHz is 25 cm. The depth of penetration at a frequency of 4 KHz will be

(a) 6.25 cm
(b) 12.50 cm
(c) 50.00 cm
(d) 100. 00 cm

[GATE 2003: 1 Mark]

Soln. **Depth of penetration** $\delta_1 = 25cm\ \ at\ \ f_1 = 1KHz$ **conductivity** σ

For a medium to be good conductor $\frac{\sigma}{\omega\epsilon} \gg 1$

$\delta = \frac{1}{\propto}$ **where** $\propto =$ **attenuation constant**

$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu + (\sigma + j\omega\epsilon)}$$

For a lossy dielectric, considered as a good conductor $\frac{\sigma}{\omega\epsilon} \gg 1$

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f\mu\sigma}$$

$$\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f\mu\sigma}}$$

$or \quad \delta\alpha\frac{1}{\sqrt{f}}$, δ_2 **be the depth of penetration at** $f_2 = 4KHz$

$$\frac{\delta_1}{\delta_2} = \sqrt{\frac{f_2}{f_1}}$$

$$\frac{\delta_1}{\delta_2} = \sqrt{\frac{4}{1}} = 2$$

$$\delta_2 = \frac{\delta_1}{2} = 12.5cm$$

Option (b)

11. The magnetic field intensity vector of a plane wave is given by $\vec{H}(x,y,z,t) = 10\sin(50000t + 0.004x + 30)\hat{a}_y\ \ where\ \ \hat{a}_y$ denotes the unit vector in y direction. The wave is propagating with a phase velocity

(a) $5 \times 10^4 m/s$
(b) $-3 \times 10^8 m/s$
(c) $-1.25 \times 10^7 m/s$
(d) $3 \times 10^8 m/s$

[GATE 2005: 1 Mark]

Soln. $H(x,y,z,t) = 10\sin[50000t + 0.004x + 30]\,\overrightarrow{a_y} = H_y\ \overrightarrow{a_y}$

$$H_y = 10\sin[\omega t + \beta x + 30]$$

$$\omega = 50{,}000\ radians\ /\ sec$$

$$\beta = -0.004\ radians\ /\ m$$

Phase velocity $V_p = \frac{\omega}{\beta} = \frac{50\times10^3}{-4\times10^{-3}}$

$$= -12.5 \times 10^6 m/sec$$

$$= -1.25 \times 10^7 m/sec$$

Represents a wave traveling in the negative x direction

Option (c)

12. The electric field of an electromagnetic wave propagating in the positive z – direction is given by
$E = \hat{a}_x \sin(\omega t - \beta z) + \hat{a}_y \sin(\omega t - \beta z + \pi/2)$
The wave is
(a) linearly polarized in the z – direction
(b) elliptically polarized
(c) left – hand circularly polarized
(d) right – hand circularly polarized

[GATE 2006: 1 Mark]

Soln. $E = \vec{a}_x \sin(\omega t - \beta z) + a_y \sin\left(\omega t - \beta z + \frac{\pi}{2}\right)$

$$= \vec{a}_x \sin(\omega t - \beta z) + a_y \cos(\omega t - \beta z)$$

$$E_x(z,t) = \sin(\omega t - \beta z)$$

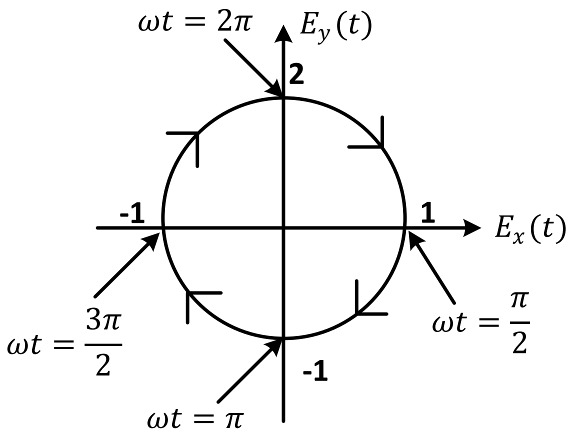
$$E_y(z,t) = \cos(\omega t - \beta z)$$

$$E_x^2 + E_y^2 = 1$$

It represents a circle in the $E_x - E_y$ plane with radius 1

The tip of the E vector is tracing the circle in the clock wise direction over a cycle from $\omega t = 0$ to 2π.

The fingers of the left hand follows the clockwise direction, thumb is pointing in the direction of propagation (+ z)



Option (c)

13. The electric field of a uniform plane electromagnetic wave in free space, along the positive X direction is given by $\vec{E} = 10(\hat{a}_y + j\hat{a}_z)e^{-j25x}$. The frequency and polarization of the wave respectively are

(a) 1.2 GHz and left circular
(b) 4 Hz and left circular
(c) 1.2 GHz and right circular
(d) 4 Hz and right circular

[GATE 2012: 1 Mark]

Soln. $\boldsymbol{E = 10(\vec{a}_y + j\vec{a}_z)e^{-j25x}}$ **in free space**

$$\vec{E} = (E_y\,\overrightarrow{a_y} + E_z\,\overrightarrow{a_z})e^{-j\beta x}$$

The wave is propagating in free space in x direction with components in y and z direction.

$$\beta = 25, \quad v_p = \frac{\omega}{\beta} = 3 \times 10^8\ m/s$$

$$\omega = 3 \times 10^8 \times 25$$

$$= 75 \times 10^8\ rad\ /\ sec$$

$$f = \frac{75\times10^8}{2\pi} = \frac{7.5\times10^9}{2\pi} \approx 1.2\ GHz$$

The field in circular polarization is found to be

$$E_s = E_o = (a_y \mp ja_z)e^{-j\beta x}$$

Propagating in +ve X direction where plus sign is used for left circular polarization and minus for right circular polarization.

Option (a)

14. A plane wave propagating in air with $\vec{E} = (8\hat{a}_x + 6\hat{a}_y + 5\hat{a}_z)e^{j(\omega t+3x-4y)} V/M$ is incident on a perfectly conducting slab positioned at $x \le 0$. The $\vec{E}$ field of the reflected wave is

(a) $(-8\hat{a}_x - 6\hat{a}_y - 5\hat{a}_z)\, e^{j(\omega t+3x+4y)} V/M$

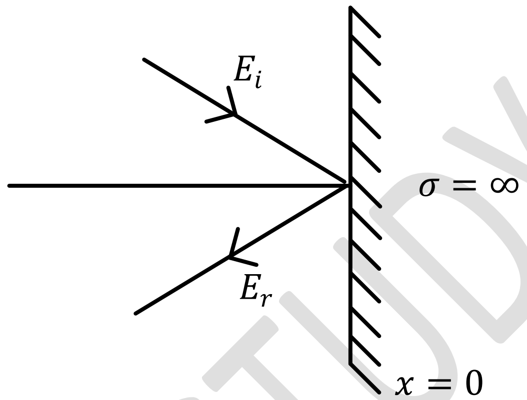
(b) $(-8\hat{a}_x + 6\hat{a}_y - 5\hat{a}_z)\, e^{j(\omega t+3x+4y)} V/M$

(c) $(-8\hat{a}_x - 6\hat{a}_y - 5\hat{a}_z)\, e^{j(\omega t-3x-4y)} V/M$

(d) $(-8\hat{a}_x + 6\hat{a}_y - 5\hat{a}_z)\, e^{j(\omega t-3x-4y)} V/M$

[GATE 2012: 1 Mark]

Soln. $\boldsymbol{E = (8a_x + 6a_y + 5a_z)e^{j(\omega t+3x-4y)}\ v/m}$



Electric field inside a perfect conductor is zero

$\boldsymbol{E_{transmitted} = 0}$

$\boldsymbol{E_i + E_r = 0}$

$\boldsymbol{E_r = -E_i = -8\hat{a}_x - 6\hat{a}_y - 8\hat{a}_z}$

The x – component of $E_{incident}$ which is normal to slab gets reflected with 180^0 phase change

Option (c)

15. A two – port network has scattering parameters given by

$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$. If the port – 2 of the two – port is short circuited, the S_{11} parameter for the resultant one – port network is

(a) $\frac{S_{11}-S_{11}S_{22}+S_{12}S_{21}}{1+S_{22}}$

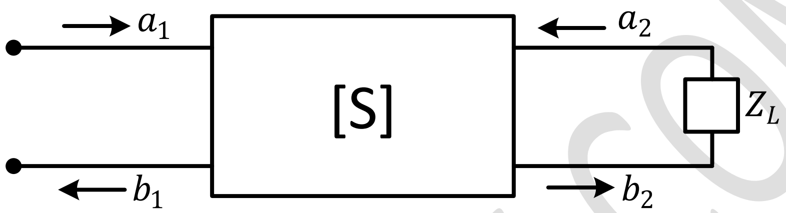
(b) $\frac{S_{11}+S_{11}S_{22}-S_{12}S_{21}}{1+S_{22}}$

(c) $\frac{S_{11}+S_{11}S_{22}+S_{12}S_{21}}{1-S_{22}}$

(d) $\frac{S_{11}-S_{11}S_{22}+S_{12}S_{21}}{1-S_{22}}$

[GATE 2014: 1 Mark]

Soln. In put reflection coefficient = T_{in}



$$Z_l = 0$$

$$b_1 = S_{11}\, a_1 + S_{12}\, a_2 ---------(I)$$

$$b_2 = S_{21}\, a_1 + S_{22}\, a_2 ---------(II)$$

$$T_{in} = \frac{b_1}{a_1} = S_{11} + S_{12}\, \frac{a_2}{a_1}$$

From (II)

$$\frac{b_2}{a_2} = S_{21}\, \frac{a_1}{a_2} + S_{22}$$

$\frac{b_2}{a_2} = \frac{1}{T_L}$ **where T_L is load reflection coefficient**

$$T_L = -1,\ Z_L = 0$$

$$or \quad \frac{1}{T_L} = S_{21}\frac{a_1}{a_2} + S_{22}$$

$$or \quad \frac{S_{22}\,T_L - 1}{T_L} = -S_{21}\, \frac{a_1}{a_2}$$

$$or \quad \frac{a_1}{a_2} = \frac{1-S_{22}\,T_L}{S_{21}\,T_L}$$

$$T_{in} = S_{11} + \frac{S_{12}\,S_{21}\,T_L}{1-S_{22}\,T_L}$$

$$T_L = -1$$

$$so, \quad T_{in} = S_{11} - \frac{S_{12}\,S_{21}}{1+S_{22}}$$

$$= \frac{S_{11}+S_{11}\,S_{22}-S_{12}\,S_{21}}{1+S_{22}}$$

Option (b)

16. Which one of the following filed patterns represents a TEM wave traveling in the positive x direction?

(a) $E = +8\hat{y}, H = -4\hat{z}$
(b) $E = -2\hat{y}, H = -3\hat{z}$
(c) $E = +2\hat{z}, H = +2\hat{y}$
(d) $E = -3\hat{y}, H = +4\hat{z}$

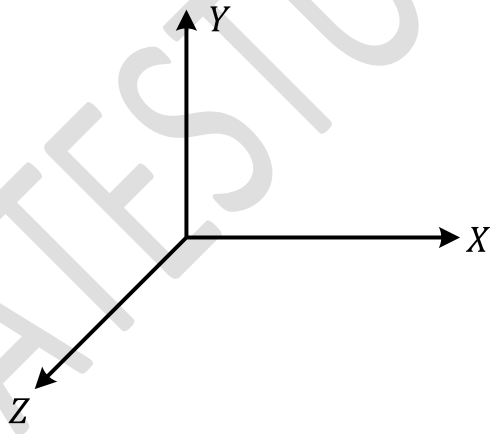
[GATE 2014: 1 Mark]

Soln. The possible combinations are

$$\hat{a}_y \times \hat{a}_z = \hat{a}_x$$

$$-\hat{a}_z \times \hat{a}_y = \hat{a}_x$$

$$-\hat{a}_y \times -\hat{a}_z = \hat{a}_x$$



Option (b)