# Plane Waves Part – II

1. For an electromagnetic wave incident from one medium to a second medium, total reflection takes place when
   (a) The angle of incidence is equal to the Brewster angle with E field perpendicular to the plane of incidence.
   (b) The angle of incidence is equal to the Brewster angle with E field parallel to the plane of incidence.
   (c) The angle of incidence is equal to the critical angle with the wave moving from the denser medium to a rarer medium
   (d) The angle of incidence is equal to the critical angle with the wave moving from a rarer medium to a denser medium

**[GATE 1987: 2 Marks]**

**Soln. For an electromagnetic wave incident from one medium to a second medium, total (internal) reflection takes place when wave moves from a denser to rarer medium and angle of incidence should be greater than or equal to the critical angle.**

**The critical angle $\boldsymbol{\theta_c}$,**

$$\sin(\theta_c) = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$$\sin(\theta_i) \geq \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

**Where $\boldsymbol{\theta_i}$ is the angle of incidence?**

**As $\sin\theta_i$ *or* $\sin\theta_c$ can not be greater than 1, $\epsilon_1 > \epsilon_2$**

**Option (c)**

2. In a good conductor the phase relation between the tangential components of electric field $E_t$ and the magnetic field $H_t$ is as follows
   (a) $E_t$ and $H_t$ are in phase
   (b) $E_t$ and $H_t$ are out of phase
   (c) $H_t$ leads $E_t$ by $90^0$
   (d) $E_t$ leads $H_t$ by $45^0$

**[GATE 1988: 2 Marks]**

**Soln. For a good conductor $\boldsymbol{\sigma \gg \omega\epsilon}$ Intrinsic impedance**

$$\eta = \frac{E}{H} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^0$$

$$\vec{E} \text{ leads } \vec{H} \text{ by } 45^0$$

**Option (d)**

3. The skin – depth of copper at a frequency of 3 GHz is 1 micron ($10^{-6}$ meter). At 12 GHz, for a non – magnetic conductor whose conductivity is 1/9 times that of copper, the skin – depth would be

(a) $\sqrt{9\times4}$ *microns*

(b) $\sqrt{9/4}$ *microns*

(c) $\sqrt{4/9}$ *microns*

(d) $\frac{1}{\sqrt{9\times4}}$ *microns*

**[GATE 1989: 2 Marks]**

**Soln. For a good conductor, skin depth** $\delta = \frac{1}{\sqrt{\pi f\mu\sigma}}$

**For copper** $\delta_1 = \frac{1}{\sqrt{\pi f_1 \mu\sigma_1}} = 10^{-6}m$

$$f_1 = 3\ GHz$$

**For non - magnetic with** $\mu = \mu_0$

$$\sigma_2 = \frac{\sigma_1}{9} \ \ at \ \ f_2 = 12\ GHz$$

$$\delta_2 = \frac{1}{\sqrt{\pi f_2 \mu \sigma_2}}$$

$$\delta \propto \frac{1}{\sqrt{f\sigma}} \quad for\ the\ same\ \mu$$

$$\frac{\delta_2}{\delta_1} = \sqrt{\frac{f_1\sigma_1}{f_2\sigma_2}} = \sqrt{\frac{3\ \sigma_1}{12\frac{\sigma_1}{9}}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$$

**Option (b)**

4. The electric field component of a uniform plane electromagnetic wave propagating in the Y – direction in a lossless medium will satisfy the equation

(a) $\frac{\partial^2 E_y}{\partial y^2} = \mu \in \frac{\partial^2 E_y}{\partial t^2}$

(b) $\frac{\partial^2 E_y}{\partial x^2} = \mu \in \frac{\partial^2 E_y}{\partial t^2}$

(c) $\frac{\partial^2 E_X}{\partial y^2} = \mu \in \frac{\partial^2 E_X}{\partial t^2}$

(d) $\frac{\sqrt{E_X^2+E_Z^2}}{\sqrt{H_X^2+H_Z^2}} = \sqrt{\frac{\mu}{\in}}$

**[GATE 1991: 2 Marks]**

**Soln.** **In a uniform plane EM wave propagating in the y – direction, the components of $\vec{E}$ and $\vec{H}$ in the direction of propagation Y $E_y,\ H_y$ are zero. $\vec{E}$ and $\vec{H}$ should be function of Y and t satisfying second order partial differential equation**

$$\frac{\delta^2 E_x}{\delta y^2} = \mu\epsilon \frac{\partial^2 E_x}{\partial t^2}$$

**$\vec{E}$ and $\vec{H}$ are related as**

$$\frac{E_z}{H_x} = \eta \quad , \quad \frac{E_x}{H_z} = -\eta$$

$$\frac{E}{H} = \sqrt{\frac{E_x^2 + E_z^2}{H_x^2 + H_z^2}} = \eta$$

$\eta$ = **Intrinsic impedance of the medium**

$$= \eta_0 = \sqrt{\frac{\mu}{\in}}$$

**Option (c) and (d)**

5. A material is described by the following electrical parameters at a frequency of 10 GHz $\sigma = 10^6 \frac{mho}{m}$, $\mu = \mu_0$ $and$ $\in/\in_0 = 10$. The material at this frequency is considered to be $\left(\in_0 = \frac{1}{36\pi} \times 10^{-9}\ F/m\right)$
   (a) Good conductor
   (b) Good dielectric
   (c) Neither a good conductor nor a good dielectric
   (d) Good magnetic material

**[GATE 1993: 2 Marks]**

**Soln. For a good conductor** $\frac{\sigma}{\omega\in} \gg 1$

$$f = 10\ GHz, \sigma = 10^6\ mho/m$$

$$\frac{\in}{\in_0} = 10 \quad or \quad \in = 10 \frac{1}{36\pi \times 10^9}$$

$$= \frac{10^{-8}}{36\pi}$$

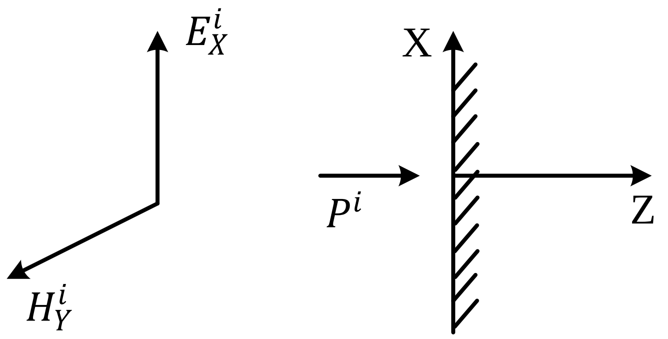
$$\frac{\sigma}{\omega\in} = \frac{10^6 \times 36\pi}{2\pi \times 10 \times 10^9 \times 10^{-8}}$$

$$= \frac{10^6 \times 36\pi \times 10^8}{2\pi \times 10^{10}}$$

$$= 18 \times 10^4 \gg 1$$

**Option (a)**

6. A plane wave is incident normally on a perfect conductor as shown in figure. Here $E_x^i$ , $H_y^i$ and $\overrightarrow{P^i}$ are electric field, magnetic field and Poynting vector respectively for the incident wave. The reflected wave should have



(a) $E_x^r = -E_x^i$

(b) $H_y^r = -H_y^i$

(c) $\overrightarrow{P^r} = -\overrightarrow{P^i}$

(d) $E_x^r = -E_X^i$

**[GATE 1993: 2 Marks]**

**Soln. The tangential component of E is continuous at the surface. That is it is the same just outside the surface as it is just inside the surface.**

**As E is zero within a perfect conductor, tangential component just outside the conductor at $z = 0, = E_x^i + E_x^r$**

**Tangential component just inside the conductor at $z = 0 += 0$**

$$E_x^r = -E_x^i$$

**The amplitude of $E_x^i$ is reversed on reflection, but $H_x^r = H_x^i$**

$$\vec{P}_r = -\vec{P}^i$$

**The average power flow is zero indicating a standing wave in the incident – reflected medium**

**Option (a) & (c)**

7. A uniform plane wave in air is normally incident on an infinitely thick slab. If the refractive index of glass slab is 1.5, then the percentage of the incident power that is reflected from the air – glass interface is

(a) 0%

(b) 4%

(c) 20%

(d) 100%

**[GATE 1996: 2 Marks]**

| **Soln.** | **Medium 1** | **Medium 2** |
|---|---|---|
| | $n_1$ | $n_2 = 1.5$ |
| | $\mu_1 = \mu_0$ | $\mu_2 = \mu_0$ |
| | $\epsilon_1 = \epsilon_0$ | $\epsilon_2 = \epsilon_0 \epsilon_r$ |

**If $n_1, n_2$ are refractive indices and $v_1, v_2$ are the velocities**

$$\frac{n_1}{n_2} = \frac{v_2}{v_1} = \frac{\sqrt{\mu_1 \epsilon_1}}{\sqrt{\mu_2 \epsilon_2}} = \frac{1}{1.5}$$

$$= \sqrt{\frac{\epsilon_1}{\epsilon_2}} \quad for \;\; \mu_1 = \mu_2 = \mu_0$$

$$\sqrt{\frac{\epsilon_1}{\epsilon_2}} = \frac{1}{1.5} = \frac{2}{3}$$

**Reflection coefficient,** $\frac{E_r}{E_i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$ $\eta_1$ $and$ $\eta_2$ **are the intrinsic impedances of medium 1 and medium 2 respectively**

$$\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}} \;\;, \;\eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}}$$

$$\mu_1 = \mu_2 = \mu_0$$

$$\frac{E_r}{E_i} = \frac{\sqrt{\epsilon_1} - \sqrt{\epsilon_2}}{\sqrt{\epsilon_1} + \sqrt{\epsilon_2}}$$

$$\frac{E_r}{E_i} = \frac{\sqrt{\frac{\epsilon_1}{\epsilon_2}} - 1}{\sqrt{\frac{\epsilon_1}{\epsilon_2}} + 1} = \frac{\frac{2}{3} - 1}{\frac{2}{3} + 1} = \frac{-\frac{1}{3}}{\frac{5}{3}} = -\frac{1}{5}$$

$$\frac{P_r}{P_i} = \frac{|E_r|^2}{|E_i|^2} = \frac{1}{25} = 4\%$$

**Option (b)**

8. Some unknown material has a conductivity of $10^6$ mho/m and a permeability of $4\pi \times 10^{-7}\ H/m$. The skin depth for the material at 1 GHz is

(a) $15.9\ \mu m$
(b) $20.9\ \mu m$
(c) $25.9\ \mu m$
(d) $30.9\ \mu m$

**[GATE 1996: 2 Marks]**

**Soln.** $\sigma = 10^6 mho/m$

$$\mu = \mu_0 = 4\pi \times 10^{-7}\ H/m$$

$$f = 1\ GHz$$

$$skin\ depth \qquad \delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\sqrt{\pi \times 10^9 \times 4\pi \times 10^{-7} \times 10^6}}$$

$$= \frac{1}{2\pi \times 10^4} m = \frac{50}{\pi}\ \mu m$$

$$= 15.9\ \mu m$$

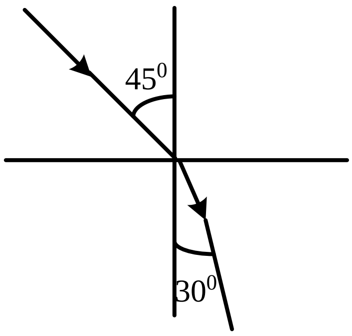
**Option (a)**

9. A uniform plane wave in air impinges at $45^0$ angle on a lossless dielectric material with dielectric constant $\varepsilon_r$. The transmitted wave propagates in a $30^0$ direction with respect to the normal. The value of $\varepsilon_r$ is

| | |
|---|---|
| (a) 1.5 | (c) 2 |
| (b) $\sqrt{1.5}$ | (d) $\sqrt{2}$ |

**[GATE 2000: 2 Marks]**

**Soln. At the interface between air and lossless dielectric angle of incidence $\theta_i = 45^0$ angle of refraction $\theta_r = 30^0$**




**According to Snell's law**

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1}$$

**$v_2$ is the velocity of wave in medium 2**

**$v_1$ is the velocity of wave in medium 1**

$$v_2 = \frac{1}{\sqrt{\mu_2 \in_2}} \quad , \quad v_1 = \frac{1}{\sqrt{\mu_1 \in_1}}$$

$$\mu_1 = \mu_2 = \mu_0 \quad , \quad \in_2 = \in_r \in_1$$

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{\sin 30^0}{\sin 45^0} = \frac{\sqrt{\mu_1 \in_1}}{\sqrt{\mu_2 \in_2}}$$

$$\frac{\sin 30^0}{\sin 45^0} = \frac{1\times\sqrt{2}}{2\times 1} = \frac{1}{\sqrt{\in_r}}$$

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{\in_r}}$$

**Option (d)**

10. A uniform plane electromagnetic wave incident normally on a plane surface of a dielectric material is reflected with a VSWS of 3. What is the percentage of incident power that is reflected?

(a) 10% (c) 50%

(b) 25% (d) 75%

**[GATE 2001: 2 Marks]**

**Soln.** **VSWR = 3**

**Let S = 3**

$$S = \frac{1+|K|}{1-|K|} \quad , \ k\ is\ the\ reflection\ coefficinet$$

$$|K| = \frac{S-1}{S+1} = \frac{3-1}{3+1} = \frac{1}{2}$$

$$\frac{E_r}{E_i} = K$$

$$\frac{P_r}{P_i} = \frac{|E_r|^2}{|E_i|^2} = K^2 = \frac{1}{4}$$

$$= 25\%$$

**Option (b)**

11. A plane wave is characterized by $\vec{E} = \left(0.5\hat{x} + \hat{y}e^{j\pi/2}\right)e^{j\omega t - jkz}$. This wave is

(a) Linearly polarized
(b) Circularly polarized
(c) Elliptically polarized
(d) Un polarized

**[GATE 2002: 2 Marks]**

**Soln.** **The wave $\vec{E} = \left(0.5\hat{x} + \hat{y}\, e^{j\pi/2}\right)e^{j\omega t - jkz}$ represents a wave travelling in the positive Z direction. Taking the real part of $\vec{E}$**

$$E_x(z,t) = 0.5\cos(\omega t - kz)$$

$$E_y(z,t) = \cos\left(\omega t + \frac{\pi}{2} - kz\right)$$

$$= -\sin(\omega t - kz)$$

$$\left(\frac{E_x}{0.5}\right)^2 + \left(\frac{E_y}{1}\right)^2 = 1$$

**This is the equation of an ellipse. The wave is elliptically polarized**

**Option (c)**

12. Distilled water at $25^0$ C is characterized by $\sigma = 1.7\times10^{-4}\,\frac{mho}{m}$ *and* $\epsilon = 78\,\epsilon_0$ at a frequency of 3 GHz. Its loss tangent $\tan\delta$ is

(a) $1.3\times10^{-5}$
(b) $1.3\times10^{-3}$
(c) $1.7\times10^{-4}/78$
(d) $1.7\times10^{-4}(78\ \epsilon_0)$

**[GATE 2002: 2 Marks]**

**Soln.** **For distilled water $\sigma = 1.7\times10^{-4}\ mho/m$**

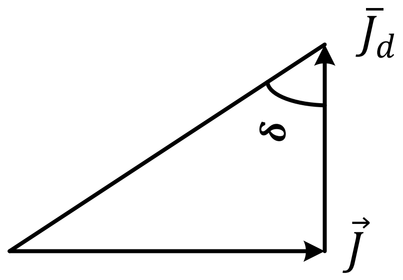
$$\epsilon = 78\,\epsilon_0 \quad , \quad f = 3\ GHz \quad , \quad \epsilon_0 = \frac{10^{-9}}{36\pi}$$

**Loss tangent** $= \tan\delta = \dfrac{\sigma}{\omega\epsilon}$

$$\frac{\sigma}{\omega\,\epsilon} = \frac{1.7\times10^{-4}\times36\pi\times10^{9}}{2\pi\times3\times10^{9}\times78}$$

$$= 0.130\times10^{-4}$$

$\cong \mathbf{1.3 \times 10^{-5}}$




$\vec{J}$ **= conduction current density =** $\boldsymbol{\sigma \vec{E}}$

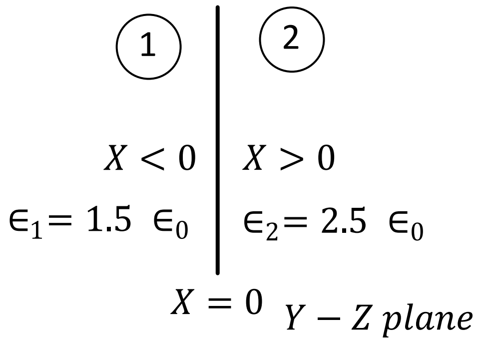
$\vec{J}_d$ **= displacement current density =** $\boldsymbol{j\omega \in \vec{E}}$

**Option (a)**

13. Medium 1 has the electrical permittivity $\varepsilon_1 = 1.5\ \varepsilon_0\ farad/m$ and occupies the region to left of $x = 0$ plane. Medium 2 has the electrical permittivity $\varepsilon_2 = 2.5\ \ \varepsilon_0$ farad / m and occupies the region to the right of $x = 0$ plane. If $E_1$ in medium 1 is $E_1 = \left(2\ u_x - 3\ u_y + 1\ u_z\right)$ volt/m, the $E_2$ in medium 2 is
(a) $(2\ u_X - 7.5\ u_Y + 2.5\ u_Z)\ \ volt/m$
(b) $(2\ u_X - 2\ u_Y + 0.6\ u_Z)\ \ volt/m$
(c) $(1.2\ u_X - 3\ u_Y + 1\ u_Z)\ \ volt/m$
(d) $(1.2\ u_X - 2\ u_Y + 0.6\ u_Z)\ \ volt/m$

**[GATE 2003: 2 Marks]**

**Soln.** **The interface between medium 1 and 2 is x = 0 or y – z plane.**




$$\boldsymbol{E_1 = \left(2\ u_x - 3\ u_y + 1\ u_z\right) volt/m}$$

**The y and z components of $E_1$ are same as the y and z components of $E_2$ as the tangential components of E are continuous.**

$$E_{t_1} = E_{t_2}$$

**The normal components of D are continuous**

$$D_{N_1} = D_{N_2}$$

$$\in_1 E_{N_1} = \in_2 E_{N_2}$$

$$\in_1 E_{X_1} = \in_2 E_{X_2}$$

$$or \quad E_{X_2} = \frac{\in_1 E_{X_1}}{\in_2} = \frac{1.5 \in_0 \times 2}{2.5 \in_0} = 1.2$$

$$\vec{E}_1 = (1.2\, u_x - 3\, u_y + 1\, u_z)\ volt/m$$

**Option (c)**

14. A uniform plane wave traveling in air is incident on the plane boundary between air and another dielectric medium with $\varepsilon_r = 4$. The reflection coefficient for the normal incidence is

(a) Zero
(b) $0.5 \angle 180^0$
(c) $0.333 \angle 0^0$
(d) $0.333 \angle 180^0$

**[GATE 2003: 2 Marks]**

**Soln. For normal incidence of a uniform plane wave at air dielectric interface, reflection coefficient** $K$, $\in_r = 4$

$$K = \frac{E_r}{E_i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$= \frac{\frac{\eta_2}{\eta_1} - 1}{\frac{\eta_2}{\eta_1} + 1}$$

$$\mu_1 = \mu_2 = \mu_0 \quad , \quad \eta_2 = \sqrt{\frac{\mu_0}{\in_2}}$$

$$\eta_1 = \sqrt{\frac{\mu_0}{\epsilon_1}}$$

$$K = \frac{\sqrt{\frac{\epsilon_1}{\epsilon_2}} - 1}{\sqrt{\frac{\epsilon_1}{\epsilon_2}} + 1}$$

**Where,** $\epsilon_1 = \epsilon_0$ **and** $\epsilon_2 = \epsilon_r \epsilon_0$

$$K = \frac{\sqrt{\frac{1}{\epsilon_r}} - 1}{\sqrt{\frac{1}{\epsilon_r}} + 1} = \frac{\frac{1}{2} - 1}{\frac{1}{2} + 1} = -\frac{1}{3}$$

$$K = -\frac{1}{3} = 0.333\angle 180^0$$

**Option (d)**

15. If the electric field intensity associated with a uniform plane electromagnetic wave traveling in a perfect dielectric medium is given by $E(z,t) = 10\cos(2\pi \times 10^7 t - 0.1\,\pi z)\ volt/m$, the velocity of the traveling wave is

(a) $3.00\times10^8\ m/sec$
(b) $2.00\times10^8\ m/sec$
(c) $6.28\times10^7\ m/sec$
(d) $2.00\times10^7\ m/sec$

**[GATE 2003: 2 Marks]**

**Soln. For a uniform plane wave in a perfect dielectric medium** $\sigma = 0$

$$E(z,t) = 10\cos(2\pi \times 10^7 t - 0.1\pi z)\ v/m$$

**The E field represents wave travelling in the positive z direction with** $\omega = 2\pi \times 10^7\ rad/\sec$ $\quad \beta = 0.1\pi\ rad/sec$

**Velocity of propagation** $V = \frac{\omega}{\beta}$

$$= \frac{2\pi \times 10^7}{0.1\pi}$$

$$= 2\times10^{8}\ m/sec$$

**Option (b)**

16. A plane electromagnetic wave propagating in free space is incident normally on a large slab of loss – less, non – magnetic, dielectric material with $\varepsilon > \varepsilon_0$. Maxima and minima are observed when the electric field is measured in front of the slab. The maximum electric field is found to be 5 times the minimum field. The intrinsic impedance of the medium should be

(a) $120\ \pi\ \Omega$ (c) $600\ \pi\ \Omega$

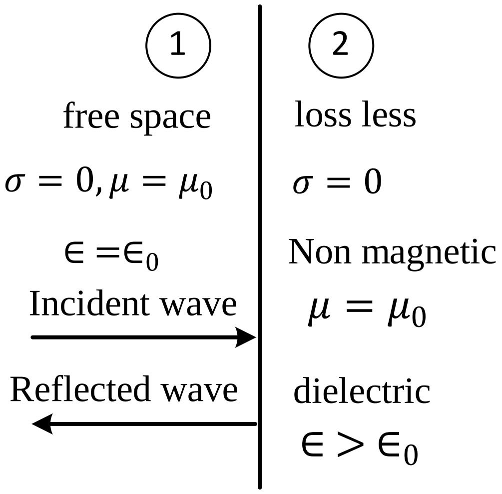
(b) $60\ \pi\ \Omega$ (d) $24\ \pi\ \Omega$

**[GATE 2004: Marks]**

**Soln.** $\boldsymbol{E_{max} = 5\ E_{min}\ \ in\ medium\ 1}$

$$\boldsymbol{VSWR, S = \frac{E_{max}}{E_{min}} = 5}$$

**Reflection coefficient** $\boldsymbol{|K| = \frac{S-1}{S+1} = \frac{5-1}{5+1} = \frac{2}{3}}$




$$\boldsymbol{K = \frac{\frac{\eta_2}{\eta_1} - 1}{\frac{\eta_2}{\eta_1} + 1} = \frac{2}{3}}$$

$$\boldsymbol{\frac{3\eta_2}{\eta_1} - 3 = \frac{2\eta_2}{\eta_1} + 2}$$

$$\frac{\eta_2}{\eta_1} = 5 \quad , \quad \eta_2 = 5\eta_1$$

$$\eta_1 = \sqrt{\frac{\mu_0}{\epsilon_0}} = \sqrt{4\pi \times 10^{-7} \times 36\pi \times 10^{9}}$$

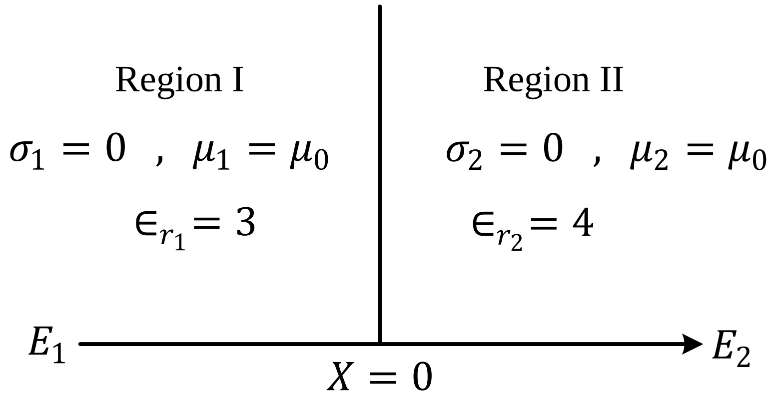
$$= (120\pi)\Omega$$

$$\eta_2 = 5 \times 120\pi$$

$$= 600\,\pi\,\Omega$$

**Option (c)**

17. A medium is divided into regions I and II about x = 0 plane, as shown in the figure below. An electromagnetic wave with electric field $\vec{E}_1 = 4\hat{a}_x + 3\hat{a}_y + 5\hat{a}_z$ is incident normally on the interface from region – I. The electric filed $E_2$ in region – II at the interface is




(a) $E_2 = E_1$
(b) $4\,\hat{a}_x + 0.7\,\hat{a}_y - 1.25\,\hat{a}_z$
(c) $3\,\hat{a}_x + 3\,\hat{a}_y + 5\,\hat{a}_z$
(d) $-3\,\hat{a}_x + 3\,\hat{a}_y + 5\,\hat{a}_z$

**[GATE 2006: 2 Marks]**

**Soln.** $\boldsymbol{E_1 = 4\hat{a}_x + 3\hat{a}_y + 5\hat{a}_z}$

**The Y and Z components of $E_1$ are same as the Y and Z components of $E_2$ as the tangential components of E are continuous at the boundary.**

$$E_{t_1} = E_{t_2}$$

$$E_{y_2} = E_{y_1} \quad , \quad E_{z_2} = E_{z_1}$$

**The normal components of D is continuous at the boundary $D_{n_1} = D_{n_2}$**

$$\epsilon_1 E_{x_1} = \epsilon_2 E_{x_2}$$

$$or \;\; E_{x_2} = \frac{\epsilon_1 E_{x_1}}{\epsilon_2} = \frac{3}{4} \times \frac{4}{1}$$

$$E_2 = 3\hat{a}_x + 3\hat{a}_y + 5\hat{a}_z$$

**Option (c)**

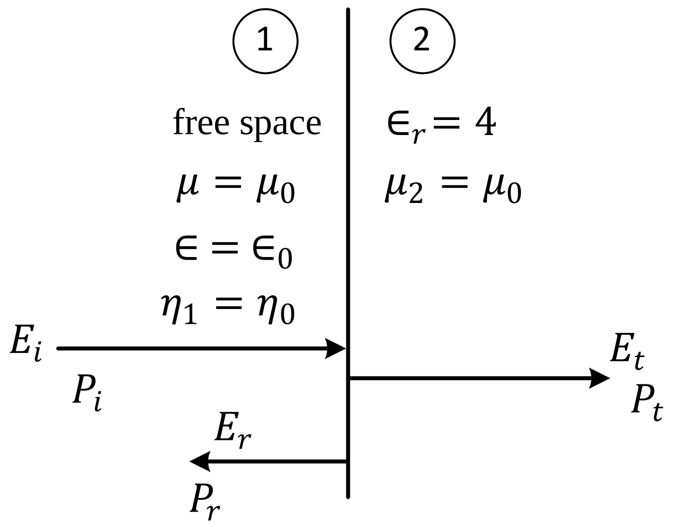
18. When a plane wave traveling in free – space is incident normally on a medium having $\varepsilon_r = 4.0$, the fraction of power transmitted into the medium is given by

(a) 8/9
(b) 1/2
(c) 1/3
(d) 5/6

**[GATE 2006: 2 Marks]**

**Soln. Normal incidence from free space on a medium is shown in figure**

$$\frac{E_t}{E_i} = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{2}{1 + \frac{\eta_1}{\eta_2}}$$



$$\frac{\eta_1}{\eta_2} = \sqrt{\frac{\mu_2 \ \epsilon_2}{\mu_1 \ \epsilon_1}} \quad \mu_1 = \mu_2 = \mu_0$$

$$= \sqrt{\epsilon_r} = 2$$

$$\frac{E_t}{E_i} = \frac{2}{1+2} = \frac{2}{3}$$

$$P_i = Incident\ power = \frac{E_i^2}{\eta_1}$$

$$P_t = Transmitted\ power = \frac{E_t^2}{\eta_2}$$

$$\frac{P_t}{P_i} = \frac{E_t^2}{E_i^2}\frac{\eta_1}{\eta_2} = \frac{4}{9} \times 2 = \frac{8}{9}$$

**Option (a)**

19. The $\vec{H}$ field (in A/m) of a plane wave propagating in free space is given by $\vec{H} = \hat{x}\frac{5\sqrt{3}}{\eta_0}\cos(\omega t - \beta z) + \hat{y}\frac{5}{\eta_0}\sin\left(\omega t - \beta z + \frac{\pi}{2}\right)$. The time average power flow density in Watts is

(a) $\frac{\eta_0}{100}$

(b) $\frac{100}{\eta_0}$

(c) $50\eta_0^2$

(d) $\frac{50}{\eta_0}$

**[GATE 2007: Marks]**

**Soln.**

$$\vec{H} = \hat{x}\frac{5\sqrt{3}}{\eta_0}\cos(\omega t - \beta z) + \hat{y}\frac{5}{\eta_0}\sin\left(\omega t - \beta z + \frac{\pi}{2}\right)$$

$$H_x = \frac{5\sqrt{3}}{\eta_0} \quad , \quad H_y = \frac{5}{\eta_0}$$

$$H_T = \sqrt{H_x^2 + H_y^2} = \sqrt{\frac{(5\sqrt{3})^2}{\eta_0^2} + \frac{5^2}{\eta_0^2}}$$

$$= \frac{5}{\eta_0}\sqrt{(\sqrt{3})^2 + 1}$$

$$= \frac{5}{\eta_0}\sqrt{4}$$

$$= \frac{10}{\eta_0}$$

**Time average power flow density**

$$P_{avg} = \frac{1}{2}\eta_0 H_T^2$$

$$= \frac{1}{2}\eta_0 \frac{100}{\eta_0^2} = \frac{50}{\eta_0}\ watts$$

**Option (d)**

20. A plane wave having the electric field component $\vec{E}_i = 24\cos(3\times10^8 t - \beta y)\,\hat{a}_z\ V/m$ and traveling in free space is incident normally on a lossless medium with $\mu = \mu_0$ and $\varepsilon = 9\,\varepsilon_0$ which occupies the region $y \geq 0$. The reflected magnetic field component is given by

(a) $\frac{1}{10\pi}\cos(3\times10^8 t + y)\,\hat{a}_x\ A/m$

(b) $\frac{1}{20\pi}\cos(3\times10^8 t + y)\,\hat{a}_x\ A/m$

(c) $-\frac{1}{20\pi}\cos(3\times10^8 t + y)\,\hat{a}_x\ A/m$

(d) $-\frac{1}{10\pi}\cos(3\times10^8 t + y)\,\hat{a}_x\ A/m$

**[GATE 2010: 2 Marks]**

**Soln.** $\quad E_{iz} = 24\cos(3\times10^8 t - \beta y)\ v/m$

$$\omega = 3\times10^8\ rad/sec$$

$$\beta = \frac{\omega}{V}$$

$$V = V_0 \;\; for\; free\; space = 3\times 10^8\; m/sec$$

$$\beta = 1\; r/m$$

$$\eta_1 = \eta_0 = 120\pi = \frac{E_{iz}}{H_{ix}}$$

$$H_{ix} = \frac{24\cos(3\times 10^8 t - y)}{120\pi} = \frac{\cos(3\times 10^8 t - y)}{5\pi}$$

$$\frac{H_r}{H_i} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2} = \frac{\frac{\eta_1}{\eta_2} - 1}{\frac{\eta_1}{\eta_2} + 1}$$

$$\frac{\eta_1}{\eta_2} = \frac{\sqrt{\mu_2 \in_2}}{\sqrt{\mu_1 \in_1}}$$

$$\mu_2 = \mu_1 = \mu_0$$

$$\frac{\eta_1}{\eta_2} = \frac{\sqrt{\in_2}}{\sqrt{\in_1}} = \sqrt{\frac{9 \in_0}{\in_0}} = 3$$

$$\frac{H_r}{H_i} = \frac{3-1}{3+1} = \frac{1}{2}$$

$$\vec{H}_r = \tfrac{1}{2}\cos(3\times 10^8 t + 1y)\,\vec{a}_x \quad A/m$$

**$\vec{H}_r$ is reflected wave which travels in negative Y direction**

**Option (a)**

21. The electric and magnetic fields for a TEM wave of frequency 14 GHz in a homogeneous medium of relative permittivity $\varepsilon_r$ and relative permeability $\mu_r = 1$ are given by

$\vec{E} = E_P \, e^{j(\omega t - 280\pi y)} \, \hat{u}_z \, V \,/\, m$

$\vec{H} = 3 \, e^{j(\omega t - 280\pi y)} \, \hat{u}_x \, V \,/\, m$

Assuming the speed of light in free space to be $3\times10^8 \, m/s$, the intrinsic impedance of free space to be 120 π, the relative permittivity $\varepsilon_r$ of the medium and the electric field amplitude $E_P$ are

(a) $\varepsilon_r = 3, E_P = 120\pi$
(b) $\varepsilon_r = 3, E_P = 360\pi$
(c) $\varepsilon_r = 9, E_P = 360\pi$
(d) $\varepsilon_r = 9, E_P = 120\pi$

**[GATE 2011: 2 Marks]**

**Soln.** $\boldsymbol{\vec{E} \; and \; \vec{H}}$ **, the wave is traveling in the Y − direction**

**Phase shift constant** $\boldsymbol{\beta = 280\pi}$

$$\boldsymbol{\eta = \frac{E}{H} = \frac{E_P}{3}}$$

**Velocity of wave** $\boldsymbol{V = \frac{\omega}{\beta}}$

$$\boldsymbol{V = \frac{2\pi\times14\times10^9}{280\pi}}$$

$$\boldsymbol{= 10^8 \; m/s}$$

$$\boldsymbol{V = \frac{1}{\sqrt{\mu \in}} = \frac{1}{\sqrt{\mu_0 \in_0}} \times \frac{1}{\sqrt{\mu_r \in_r}}}$$

**Velocity for free space** $\boldsymbol{\frac{1}{\sqrt{\mu_0 \in_0}} = 3\times10^8 \; m/sec}$

$$\boldsymbol{V = 3\times10^8 \times \frac{1}{\sqrt{\mu_r \in_r}}}$$

$$\boldsymbol{\mu_r = 1}$$

$$\boldsymbol{V = \frac{3\times10^8}{\sqrt{\in_r}} = 10^8 \quad , \quad \in_r = 9}$$

$$\boldsymbol{\eta = \sqrt{\frac{\mu}{\in}} = \sqrt{\frac{\mu_r \mu_0}{\in_r \in_0}} = \sqrt{\frac{\mu_0}{\in_0}} \times \frac{1}{\sqrt{\in_r}}}$$

$$\mu_r = 1$$

$$\eta = \frac{120\pi}{\sqrt{\epsilon_r}}$$

$$\frac{E_P}{3} = \frac{120\pi}{\sqrt{\epsilon_r}}$$

$$E_P = 120\pi\ V/m$$

**Option (c)**

22. If the electric field of a plane wave is
$\vec{E}(z,t) = \hat{x}3\cos(\omega t - kz + 30^0) - \hat{y}4\sin(\omega t - kz + 45^0)\ (mV/m)$
The polarization state of the plane wave is
(a) Left elliptical
(b) Left circular
(c) Right elliptical
(d) Right circular

**[GATE 2014: 2 Mark]**

**Soln. Equation for electric field of plane wave is**

$$\vec{E}(z,t) = \hat{x}.3\cos(\omega t - kz + 30^0) - \hat{y}.4\sin(\omega t - kz + 45^0)\ (mV/m)$$

**Here wave is propagation in +z direction. Above equation can be written for z = 0**

$$\vec{E}(0,t) = \hat{x}.3\cos(\omega t + 30^0) - y.4\sin(\omega t + 45^0)$$

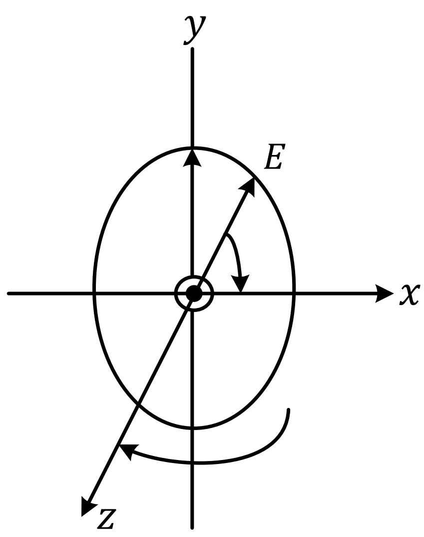
$$= \hat{x}.3\cos(\omega t + 30^0) + y.4\sin(\omega t + 45^0 + 180^0)$$

$$E(0,t) = \hat{x}.3\cos(\omega t + 30^0) + \hat{y}.4\cos(\omega t + 135^0)$$

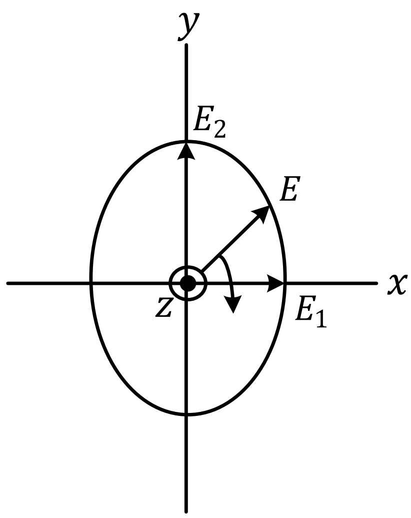
**$E_y$ component is leading $E_x$ by $105^0$**

**As in the given figure the resultant E vector rotates in clockwise direction as shown. Since the direction of propagation is +z direction.**

**If we look the wave in +z direction then it will look moving counter clockwise or left hand elliptically polarized $\left(|E_x| \neq |E_y|\right)$**




**Hint: For right handed coordinate system. In the given figure**




**$E_1$ - amplitude of wave linearly polarized in x direction**

**$E_2$ - amplitude of wave linearly polarized in y direction**

**E - Resultant vector**

**If $E_2$ leads $E_1$, then E vector moves (clockwise) as indicated**

**If $E_1$ leads $E_2$ then it moves in counter clockwise. To observe the wave polarization, we have to look towards the direction of propagation of wave.**

**(Refer: Antennas by Kraus)**

23. The electric field of a plane wave propagation in a lossless non – magnetic medium is given by the following expression

$$\vec{E}(z,t) = \hat{a}_x 5\cos(2\pi\times10^9 t + \beta z) + \hat{a}_y 3\cos\left(2\pi\times10^9 t + \beta z - \frac{\pi}{2}\right)$$

The type of the polarization is

(a) Right hand circular
(b) Left hand elliptical
(c) Right hand elliptical
(d) Linear

**[GATE 2015: 2 Mark]**

**Soln. Given**

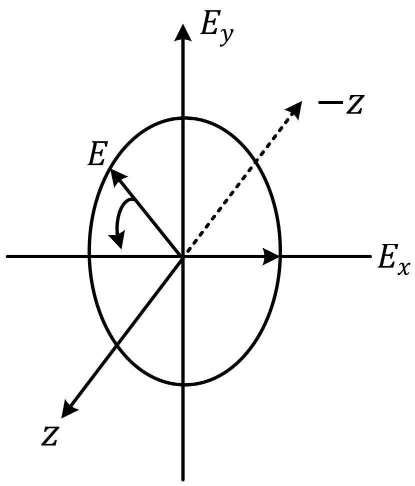
$$\vec{\boldsymbol{E}}(\boldsymbol{z},\boldsymbol{t}) = \hat{\boldsymbol{a}}_x \mathbf{5}\cos(\mathbf{2}\boldsymbol{\pi}\times\mathbf{10^9}\boldsymbol{t} + \boldsymbol{\beta z}) + \hat{\boldsymbol{a}}_y\mathbf{3}\cos\left(\mathbf{2}\boldsymbol{\pi}\times\mathbf{10^9}\boldsymbol{t} + \boldsymbol{\beta z} - \frac{\boldsymbol{\pi}}{\mathbf{2}}\right)$$

**Let** $\boldsymbol{\omega} = \mathbf{2}\boldsymbol{\pi}\times\mathbf{10^9}$

$$\vec{\boldsymbol{E}}(\boldsymbol{z},\boldsymbol{t}) = \hat{\boldsymbol{a}}_x \mathbf{5}\cos(\boldsymbol{\omega t} + \boldsymbol{\beta z}) + \hat{\boldsymbol{a}}_y\mathbf{3}\cos\left(\boldsymbol{\omega t} + \boldsymbol{\beta z} - \frac{\boldsymbol{\pi}}{\mathbf{2}}\right)$$

**Here,** $\mathbf{E_y}$ **is lagging** $\mathbf{E_x}$ **by** $\mathbf{90^0}$

**The resultant vector moves as shown (opposite to leading case) as shown in the given figure**




**Direction of propagation as per the equation is –z direction**

**Thus, the resultant vector E moves in counter clock clockwise if we look into –z direction**

**Thus, the wave is left hand elliptically polarized (since unequal amplitude)**

**Option (b)**