Transmission Lines

1. A load impedance, (200 + j0) Ω is to be matched to a 50 Ω lossless transmission line by using a quarter wave line transformer (QWT). The characteristic impedance of the QWT required is _________

[GATE 1994: 1 Mark]

Soln. For Quarter wave line transformer

$$\boldsymbol{Z_0^2 = Z_{in}.Z_L}$$

$$\boldsymbol{Z_0^2 = 50 \times 200}$$

$$\boldsymbol{Z_0 = 100\ \Omega}$$

2. A lossless transmission line having 50 Ω characteristic impedance and length $\lambda/4$ is short circuited at one end and connected to an ideal voltage source of 1V at the other end. The current drawn from the voltage sources is

(a) 0
(b) 0.02 A
(c) ∞
(d) None of the these

[GATE 1996: 1 Mark]

Soln. For quarter wave transformer $(\lambda/4)$

$$\boldsymbol{Z_{in} = \frac{Z_0^2}{Z_L}}$$

$$\boldsymbol{Z_L = 0} \quad \textbf{(short circuit)}$$

$$\boldsymbol{Z_{in} = \frac{Z_0^2}{0} = \infty} \quad \textbf{(open circuit)}$$

The current drawn from the voltage source $\boldsymbol{I_S = \frac{V_S}{Z_{in}} = \frac{V_S}{\infty} = 0}$

Option (a)

3. The capacitance per unit length and the characteristic impedance of a lossless transmission line are C and Z_0 respectively. The velocity of a travelling wave on the transmission line is

(a) Z_0C
(b) $1/(Z_0C)$
(c) Z_0/C
(d) C/Z_0

[GATE 1996: 1 Mark]

Soln. $\boldsymbol{Z_0 = \sqrt{\frac{L}{C}}}$, $\boldsymbol{Z_0^2 = \frac{L}{C}}$

$$\boldsymbol{velocity\ (V) = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(Z_0^2C)(C)}} = \frac{1}{Z_0C}}$$

Option (b)

4. A transmission line of 50 Ω characteristic impedance is terminated with a 100 Ω resistance. The minimum impedance measured on the line is equal to
(a) 0 Ω
(b) 25 Ω
(c) 50 Ω
(d) 100 Ω

[GATE 1997: 1 Mark]

Soln. $\boldsymbol{Z_0 = 50\Omega}$

$\boldsymbol{Z_L = 100\Omega}$

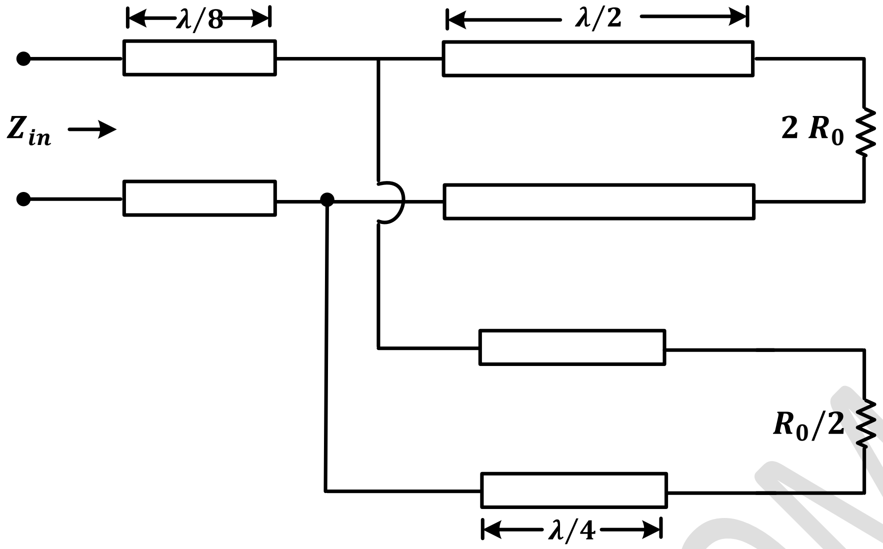
$\boldsymbol{Z_L > Z_0}$

$$\boldsymbol{Z_{in(min)} = \frac{Z_0^2}{Z_L}}$$

$$\boldsymbol{= \frac{50\times 50}{100} = 25\Omega}$$

Option (b)

5. All transmission line section in Figure, have a characteristic impedance $R_0 + j0$. The input impedance Z_{in} equals



(a) $\frac{2}{3}R_0$ (c) $\frac{3}{2}R_0$
(b) R_0 (d) $2\ R_0$

[GATE 1998: 1 Mark]

Soln. **For $\lambda/4$ line,** $Z_{in_1} = \frac{Z_0^2}{Z_L} = \frac{R_0^2}{R_0/2} = 2R_0$

For $\lambda/2$ line, $Z_{in_2} = Z_{L_2} = 2R_0$

For $\lambda/8$ line, $Z_L = (2R_0) \parallel 2R_0$

$= R_0$

For transmission line of length $l\,, Z_{in} = Z_0\left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right]$

$$l = \lambda/8, \qquad Z_{in} = Z_0\left[\frac{Z_L + jZ_0}{Z_0 + jZ_L}\right]$$

$$Z_{in} = R_0\left[\frac{R_0 + jR_0}{R_0 + jR_0}\right] = R_0$$

Option (b)

6. The magnitudes of the open – circuit and short – circuit input impedances of a transmission line are 100 Ω and 25 Ω respectively. The characteristic impedance of the line is.

(a) 25 Ω (c) 75 Ω
(b) 50 Ω (d) 100 Ω

[GATE 2000: 1 Mark]

Soln. $Z_0 = \sqrt{Z_{0C} . Z_{SC}} = \sqrt{100 \times 25}$

$$Z_0 = 10 \times 5$$

$$= 50\Omega$$

Option (b)

7. A transmission line is distortion less if

(a) $RL = \frac{1}{RC}$ (c) $RL = RC$
(b) $RL = GC$ (d) $RL = LC$

[GATE 2001: 1 Mark]

Soln. For a distortion less line, velocity of propagation $v = \frac{\omega}{\beta}$ must be independent of frequency. To achieve this

$$LG = CR$$

$$or \quad \frac{L}{C} = \frac{R}{G}$$

Option (c)

8. The VSWR can have any value between

(a) 0 and 1 (c) 0 and ∞
(b) – 1 and + 1 (d) 1 and ∞

[GATE 2002: 1 Mark]

Soln. $VSWR = \frac{1+|\rho|}{1-|\rho|}$

Where ρ is reflection coefficient ρ can take values between 0 and 1 when $\rho = 0,\ VSWR = 1$

$$\rho = 1,\ VSWR = \infty$$

Option (d)

9. A transmission line has a characteristic impedance of 50Ω and a resistance of 0.1 Ω/m. If the line is distortion less, the attenuation constant (in Np/m) is

(a) 500
(b) 5
(c) 0.014
(d) 0.002

[GATE 2010: 1 Mark]

Soln. **Attenuation constant α to be independent of frequency for distortion less transmission** $\alpha = \sqrt{RG}$

For distortion less transmission:

$$\frac{L}{C} = \frac{R}{G}$$

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{R}{G}}$$

$$\alpha = \sqrt{RG} = \sqrt{R}\frac{\sqrt{R}}{Z_0}$$

$$= \frac{R}{Z_0}$$

$$= \frac{0.1}{50}$$

$$= 0.002\ Np/m$$

Option (d)

10. A transmission line of characteristic impedance 50Ω is terminated by a 50Ω load. When excited by a sinusoidal voltage source at 10 GHz, the phase difference between two points spaced 2 mm apart on the line is found to be π/4 radians. The phase velocity of the wave along the line is

(a) $0.8 \times 10^8\ m/s$
(b) $1.2 \times 10^8\ m/s$
(c) $1.6 \times 10^8\ m/s$
(d) $3 \times 10^8\ m/s$

[GATE 2011: 1 Mark]

Soln. **Phase difference** $\beta l = \frac{2\pi}{\lambda}$ **path difference**

$$\frac{\pi}{4} = \frac{2\pi}{\lambda}(2 \times 10^{-3})$$

$\lambda = 8 \times 2 \times 10^{-3}$

$= 16 \times 10^{-3}\ m$

Given , $f = 10GHz$

The phase velocity of the wave:

$V_p = f\lambda$

$= 10 \times 10^{9} \times 16 \times 10^{-3}$

$= 160 \times 10^{6}\ m/sec$

$= 1.6 \times 10^{8}\ m/sec$

Option (c)

11. The return loss of a device is found to be 20 dB. The voltage standing wave ratio (VSWR) and magnitude of reflection coefficient are respectively

(a) 1.22 and 0.1
(b) 0.81 and 0.1
(c) – 1.22 and 0.1
(d) 2.44 and 0.2

[GATE 2013: 1 Mark]

Soln. Return loss (dB) = $-20log_{10}|\rho|$

Where ρ **is the reflection coefficient .**

For $|\rho| = 1$ **full reflection**

Return Loss = 0 dB

If $|\rho| = 0.1$

$R.Loss\ (dB) = -20\ log_{10}(0.1)$

$= -20 \times (-1)$

= 20 dB

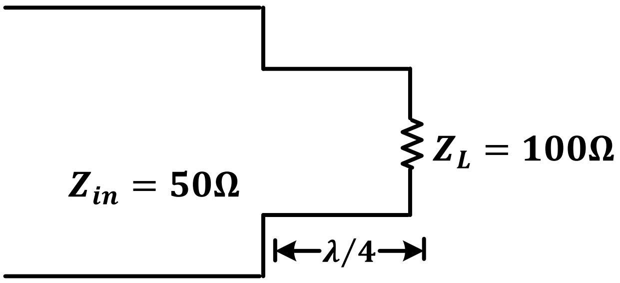
$$VSWR = \frac{1+|\rho|}{1-|\rho|}$$

$$= \frac{1+0.1}{1-0.1} = \frac{1.1}{0.9} = 1.22$$

Option (a)

12. To maximize power transfer, a lossless transmission line is to be matched to a resistive load impedance via a λ/4 transformer as shown. The characteristic impedance (in Ω) of the λ/4 transformer is ______.

Soln. Input impedance for quarter wave transfer



$$Z_{in} = \frac{Z_0^2}{Z_L}$$

$$Z_0^2 = Z_{in}\, Z_L$$

$$Z_0 = \sqrt{Z_{in}\, Z_L}$$

$$= \sqrt{50 \times 100}$$

$$= 70.72\ \Omega$$

Two Marks Questions

1. A transmission line of pure resistive characteristic impedance is terminated with an unknown load. The measured value of VSWR on the line is equal to 2 and a voltage minimum point is found to be at the load. The load impedance is then

(a) Complex
(b) Purely capacitive
(c) Purely resistive
(d) Purely inductive

[GATE 1987: 2 Marks]

Soln. If V_{min} or V_{max} Occurs at the load for a lossless transmission line then load impedance Z_L is purely resistive

Option (c)

2. A two – wire transmission line of characteristic impedance Z_0 is connected to a load of impedance $Z_L (Z_L \neq Z_0)$. Impedance matching cannot be achieved with
(a) A quarter – wavelength transformer
(b) A half – wavelength transformer
(c) An open – circuited parallel stub
(d) A short – circuited parallel stub

[GATE 1988: 2 Marks]

Soln. If $\boldsymbol{Z_L \neq Z_0}$

Then, impedance matching can be achieved by

(i) a quarter wavelength transformer $(\lambda/4)$.
(ii) an open – circuited parallel stub.
(iii) a short – circuited parallel stub.

Half wave length transformer $(\lambda/2)$ cannot be used for impedance matching

Option (b)

3. A 50 ohm lossless transmission line has a pure reactance of (j 100) ohms as its load. The VSWR in the line is
(a) 1/2
(b) 2
(c) 4
(d) (infinity)

[GATE 1989: 2 Marks]

Soln. Reflection coefficient

$$\boldsymbol{\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{j\,100 - 50}{j\,100 + 50}}$$

$$\boldsymbol{\Gamma = \frac{\sqrt{100^2 + 50^2}}{\sqrt{100^2 + 50^2}} = 1}$$

$$\boldsymbol{VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 1}{1 - 1} = \frac{2}{0} = \infty}$$

Option (d)

4. The input impedance of a short circuited lossless transmission line quarter wave long is
(a) Purely reactive
(b) Purely resistive
(c) Infinite
(d) Dependent on the characteristic impedance of the line

[GATE 1991: 2 Marks]

Soln. For a quarter wave line

$$Z_{in} = \frac{Z_0^2}{Z_L}$$

$$Z_L = 0$$

$$Z_{in} = \frac{Z_0^2}{0} = \infty$$

Option (c)

5. A transmission line whose characteristic impedance is a pure resistance
(a) Must be a lossless line
(b) Must be a distortion less line
(c) May not be a lossless line
(d) May not be a distortion less line

[GATE 1992: 2 Marks]

Soln. If the transmission line is to have neither frequency nor delay distortion, then α (attenuation constant) and velocity of propagation cannot be functions of frequency.

$$v = \frac{\omega}{\beta}$$

β must be a direct function of frequency to achieve this condition

$$LG = CR$$

$$\frac{L}{C} = \frac{R}{G}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

For a lossless line, $Z_0 = \sqrt{\frac{L}{C}}$

$$\alpha = \sqrt{RG} = 0 \; for \; R = 0, G = 0$$

$$\beta = \omega\sqrt{LC}$$

A loss less line is always a distortion less line

6. Consider a transmission line of characteristic impedance 50 ohms. Let it be terminated at one end by (+ j50) ohm. The VSWR produced by it in the transmission line will be
   (a) + 1
   (b) 0
   (c) ∞
   (d) + j

[GATE 19993: 2 Marks]

Soln. Reflection coefficient $= \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$

$$\Gamma = \frac{j50 - 50}{j50 + 50} = \frac{-50 + j50}{50 + j50}$$

$$\Gamma = \frac{\sqrt{50^2 + 50^2}}{\sqrt{50^2 + 50^2}} = 1$$

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 1}{1 - 1} = \frac{2}{0} = \infty$$

Option (c)

7. If a pure resistance load, when connected to a lossless 75 ohm line, produce a VSWR of 3 on the line, then the load impedance can only be 25 ohms. True/False

[GATE 1994: 2 Marks]

Soln. On a lossless line of $R_0 = 75\Omega$ with resistance load R_L

VSWR = S = 3

$$S = \frac{R_L}{R_0} \; if \; R_L > R_0$$

$$= \frac{R_0}{R_L} \; if \; R_L < R_0$$

$$R_L = SR_0 = 3 \times 75$$

$$= 225\ \Omega$$

$$R_L = \frac{R_0}{S} \; if \; R_L < R_0$$

$$= \frac{75}{3} = 25\Omega$$

$$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|} \quad , \quad \Gamma = \frac{R_L - R_0}{R_L + R_0}$$

$$for \;\; R_0 = 75\Omega\,, R_L = 25\Omega$$

$$\Gamma = \frac{25-75}{25+75} = \frac{-50}{100} = -\frac{1}{2}$$

$$For \;\; R_0 = 75\Omega, R_L = 225\Omega$$

$$\Gamma = \frac{R_L - R_0}{R_L + R_0} = \frac{150}{300} = \frac{1}{2}$$

$$|\Gamma| = \frac{1}{2} \text{ in either case and } S = \frac{1+\frac{1}{2}}{1-\frac{1}{2}} = 3$$

The statement, the load impedance can only be 25Ω is FALSE

8. In a twin – wire transmission line in air, the adjacent voltage maximum are at 12.5cm and 27.5cm. The operating frequency is
   (a) 300 MHz
   (b) 1 GHz
   (c) 2 GHz
   (d) 6.28 GHz

[GATE 1999: 2 Marks]

Soln. Distance between adjacent voltage maximum = $\lambda/2$

$$\lambda/2 = 27.5 - 12.5$$

= 15 cm

$$\lambda = 30\ cm$$

Velocity of propagation on twin – wire T_L line

$$v = 3 \times 10^8 m/sec$$

$$f = \frac{v}{\lambda} = \frac{3 \times 10^8}{30 \times 10^{-2}}$$

$$= \frac{3 \times 10^{10}}{30} = \frac{30 \times 10^9}{30} Hz$$

= 1 GHz

Option (b)

9. In air, a lossless transmission line of length 50 cm with $L = 10\mu H/m, C = 40PF/m$ is operated at 25 MHz. It's electrical path length is
   (a) 0.5 meters
   (b) λ meters
   (c) $\frac{\pi}{2}$ radians
   (d) 180 degrees

[GATE 1999: 2 Marks]

Soln. Electrical path length = $\beta\ l$ radians

$$velocity\ \ v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10 \times 10^{-6} \times 40 \times 10^{-12}}}$$

$$= \frac{1}{10^{-9} \times 20} = 0.5 \times 10^8\ m/s$$

$$v = 0.5 \times 10^8\ m/s$$

$$\lambda = \frac{v}{f} = \frac{0.5 \times 10^8}{25 \times 10^6} = \frac{50}{25}$$

= 2 meters

$$\beta l = \frac{2\pi}{\lambda} \times l$$

$$= \frac{2\pi}{2} \times \frac{50}{100}$$

$$\frac{\pi}{2} \; radians$$

Option (c)

10. A uniform plane electromagnetic wave incident normally on a plane surface of a dielectric material is reflected with a VSWR of 3. What is the percentage of incident power that is reflected

(a) 10%
(b) 25%
(c) 50%
(d) 75%

[GATE 2001: 2 Marks]

Soln.

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$3 = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$\Gamma = 0.5$$

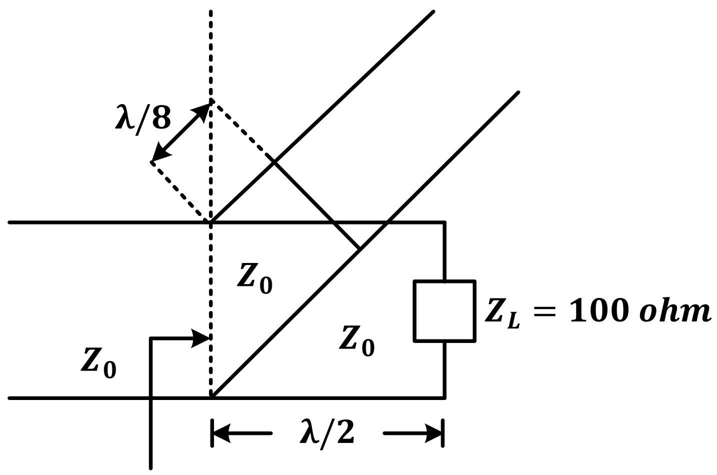
$$\frac{P_r}{P_i} = \Gamma^2 = (0.5)^2$$

= 0.25

25% of incident power is reflected.

Option (b)

11. A short circuited stub is shunt connected to a transmission line as shown in figure. If $Z_0 = 50\Omega$, the admittance Y seen at the junction of the stub and transmission line is



(a) (0.01 – j 0.02) mho
(b) (0.02 – j 0.01) mho
(c) (0.04 + j 0.02) mho
(d) (0.02 + j 0) mho

[GATE 2003: 2 Marks]

Soln. For both Transmission line and stub, $Z_0 = 50\Omega$

For $\lambda/2$ line input impedance $Z_{il} = Z_L$

$$Z_{il} = Z_L = 100\Omega$$

$$Y_{il} = 0.01\ mho$$

For short circuited stub input impedance

$$Z_{i2} = jZ_0\ tan(\beta l)$$

$$= j\ Z_0\ tan\left(\frac{2\pi}{\lambda}\frac{\lambda}{8}\right)$$

$$= j\ Z_0\ tan\left(\frac{\pi}{4}\right)$$

$$= j\ Z_0 = j\ 50$$

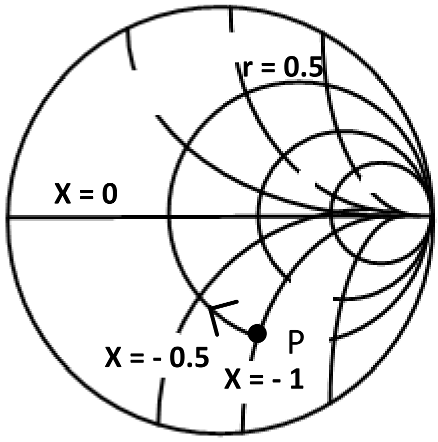
$$Y_{i2} = \frac{1}{j\ 50} = -j\ 0.02$$

$$Y = Y_{il} + Y_{i2}$$

= (0.01 – j 0.02) mho

Option (a)

12. Consider an impedance $Z = R + jX$ marked with point P in an impedance smith chart as shown in figure. The movement from point P along a constant resistance circle in the clockwise direction by an angle 45^0 is equivalent



(a) Adding an inductance in series with Z
(b) Adding a capacitance in series with Z
(c) Adding an inductance in shunt across Z
(d) Adding a capacitance in shunt across Z

[GATE 2004: 2 Marks]

Soln. **Point P ($Z = R + jX$) on the Smith chart as shown in figure is the intersection of constant resistance circle $r = 0.5$ and constant reactance circle $X = -1$, Normalized impedance $Z = 0.5 - j1$**

The movement from point P along constant resistance circle of 0.5 by 45^0 in clockwise direction, resistance 0.5 is not changed but positive reactance is added. This is equivalent to adding inductance in series with Z.

Option (a)

13. Characteristic impedance of a transmission line is 50Ω. Input impedance of the open circuited line is $Z_{0C} = 100 + j\ 150\Omega$. When the transmission line is short circuited then the value of the input impedance will be

(a) 50Ω
(b) $100 + j\ 50\Omega$
(c) $7.69 + j\ 11.54\Omega$
(d) $7.69 - j\ 11.54\Omega$

[GATE 2005: 2 Marks]

Soln.

$$Z_0 = \sqrt{Z_{OC}\ \ Z_{SC}}$$

$$Z_O^2 = Z_{OC}\ \ Z_{SC}$$

$$Z_{SC} = \frac{Z_O^2}{Z_{OC}}$$

$$= \frac{50 \times 50}{100 + j\,150} = \frac{50}{2 + j3}$$

$$= \frac{50(2 - 3j)}{4 + 9}$$

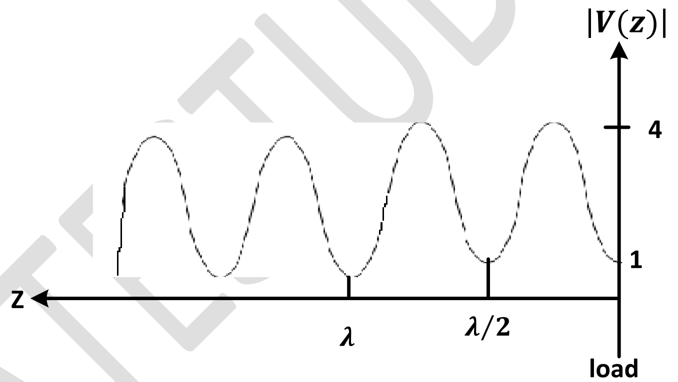
$$= \frac{100 - 50j}{13}$$

$$= 7.69 - j\,11.54$$

Option (d)

Common data for Question 14 and 15.

Voltage standing wave pattern in a impedance 50Ω and a resistive load is shown in the figure.



14. The value of the load resistance is

(a) 50Ω
(b) 200Ω
(c) 12.5Ω
(d) 0Ω

[GATE 2005: 2 Marks]

Soln.

$$VSWR(S) = \frac{V_{max}}{V_{min}} = \frac{4}{1}$$

S = 4

$$Z_{max} = Z_O S$$

$$Z_{min} = \frac{Z_O}{S}$$

As minima is at load

$$Z_L = Z_{min} = \frac{Z_O}{S}$$

$$Z_L = \frac{50}{4} = 12.5\Omega$$

Option (c)

15. The reflection coefficient is given by
    (a) – 0.6
    (b) – 1
    (c) 0.6
    (d) 0

[GATE 2005: 2 Marks]

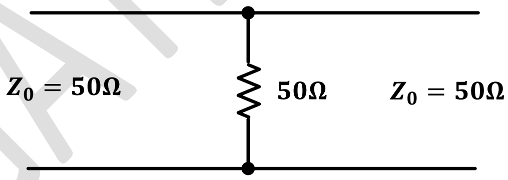
Soln. The reflection coefficient

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma = \frac{12.5 - 50}{12.5 + 50} = -0.6$$

Option (a)

16. A load of 50Ω is connected in shunt in a 2 – wire transmission line of $Z_0 = 50\Omega$ as shown in the figure. The 2 – port scattering parameter (s – matrix) of the shunt element is



(a) $\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} -\frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$

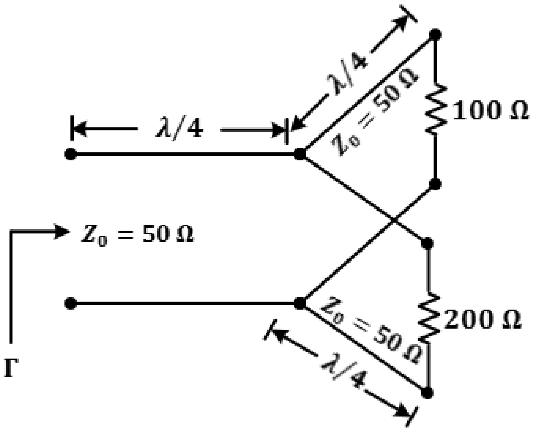
(d) $\begin{bmatrix} \frac{1}{4} & -\frac{3}{4} \\ \frac{1}{2} & \frac{1}{4} \end{bmatrix}$

[GATE 2007: 2 Marks]

Soln. **The line is terminated with 50 ohms at the ends, so matched on both the sides thus $S_{11} = 0, S_{22} = 0$ and $S_{12} = S_{21} = 1$**

Option (b)

17. The parallel branches of a 2 – wire transmission line are terminated in 100 Ω and 200 Ω resistors as shown in the figure. The characteristic impedance of the line is 50 and each section has a length of $\frac{\lambda}{4}$. The voltage reflection coefficient Γ at the input is



(a) $-j\,\frac{7}{5}$

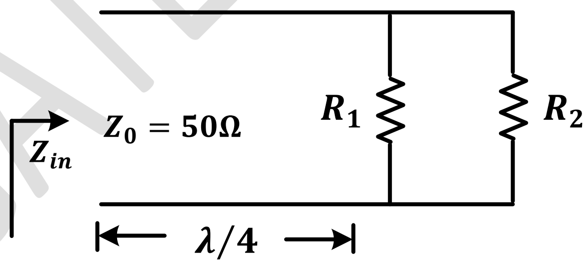
(b) $-\frac{5}{7}$

(c) $j\,\frac{5}{7}$

(d) $\frac{5}{7}$

[GATE 2007: 2 Marks]

Soln.



$$Z_{in} = \frac{Z_0^2}{Z_L} \quad for\ \lambda/4 \quad line$$

$$R_1\ due\ to\ 100\ \Omega = \frac{50^2}{100}$$

$$= 25\ \Omega$$

$$R_2 \ \text{due to}\ 200\ \Omega = \frac{50^2}{200}$$

$$= \frac{25}{2}\ \Omega$$

$$R_1 \parallel R_2 = 25 \ \Big\| \frac{25}{2} = \frac{25}{3}$$

$$Z_{in} = \frac{Z_0^2}{Z_L}$$

$$= \frac{50 \times 50}{25/3} = 300\Omega$$

Reflection coefficient

$$\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

$$\Gamma = \frac{300 - 50}{300 + 50}$$

$$= \frac{5}{7}$$

Option (d)

18. One end of a lossless transmission line having the characteristic impedance of 75 and length of 1 cm is short circuited. At 3 GHz, the input impedance at the other end of the transmission line is

(a) 0
(b) Resistive
(c) Capacitive
(d) Inductive

[GATE 2008: 2 Marks]

Soln. $f = 3GHz$

$$\beta = \frac{2\pi}{\lambda}$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{3 \times 10^9} = \frac{1}{10}\ m$$

$$\beta l = 2\pi \times 10 \times \frac{1}{100}$$

$$= \frac{\pi}{5}$$

$$= 36^0$$

Input impedance of short circuited line

$$Z_{in} = j\, Z_0\, tan\, \beta l$$

$$= j\, Z_0\, tan\, 36^0$$

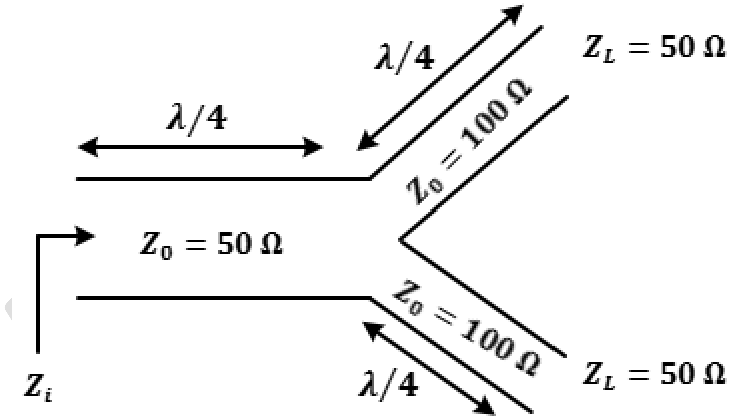
$$= j\, 75\, tan\, 36^0$$

$$= j\, 54.49\, \Omega$$

Input impedance is inductive

Option (d)

19. A transmission line terminates in two branches each of length $\frac{\lambda}{4}$ as shown. The branches are terminated by 50 Ω loads. The lines are lossless and have the characteristic impedances shown. Determine the impedance Z_i as seen by the source



(a) 200 Ω
(b) 100 Ω
(c) 50 Ω
(d) 25 Ω

[GATE 2009: 2 Marks]

Soln. For a $\frac{\lambda}{4}$ line of characteristic impedance Z_0 and terminated by Z_L , input impedance

$$Z_1 = \frac{Z_0^2}{Z_L}$$

$$Z_1 = \frac{Z_0^2}{Z_{L_1}} = \frac{100^2}{50} = 200\Omega$$

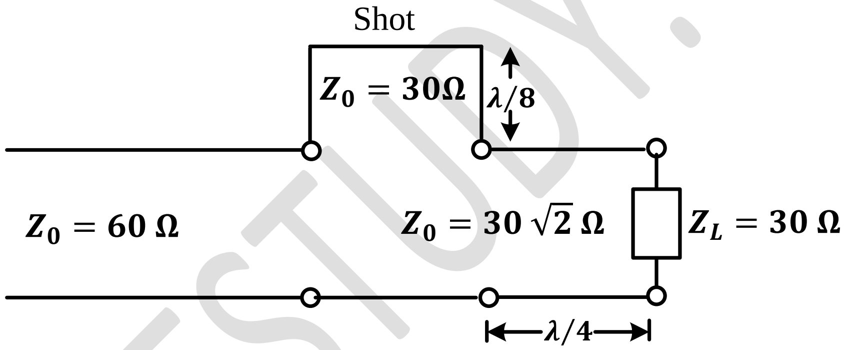
$$Z_2 = \frac{Z_0^2}{Z_{L_2}} = \frac{100^2}{50} = 200\Omega$$

$$Z_L = Z_1 \| Z_2 \quad = \quad 200 \| 200 = 100\Omega$$

$$Z_i = \frac{Z_0^2}{Z_L} = \frac{50^2}{100} = 25\Omega$$

Option (d)

20. In the circuit shown, all the transmission line sections are lossless. The voltage standing wave ratio (VSWR) on the line



(a) 1.00
(b) 1.64
(c) 2.50
(d) 3.00

[GATE 0000: 2 Marks]

Soln. **The input impedance of a transmission line of length l of characteristic impedance Z_0 and terminated by load Z_L**

$$Z_{in} = Z_0 \left(\frac{Z_L + j\, Z_0 \tan \beta l}{Z_0 + j\, Z_L \tan \beta l} \right)$$

Input impedance of shorted $\frac{\lambda}{8}$ line of $Z_0 = 30\ \Omega$

$$Z_i = 30\left(\frac{0 + j\,30\,tan\,\left(\frac{2\pi}{\lambda}\right)\frac{\lambda}{8}}{30 + 0}\right)$$

$$Z_i = j\,30\,tan\,\frac{\pi}{4} = j\,30$$

Input impedance of $\frac{\lambda}{4}$ line of $Z_0 = 30\sqrt{2}\,\Omega$ and $Z_L = 30\Omega$

$$Z_2 = \frac{Z_0^2}{Z_L}$$

$$= \frac{\left(30\sqrt{2}\right)^2}{30} = \frac{30\sqrt{2} \times 30\sqrt{2}}{30}$$

$$= 60\,\Omega$$

Load impedance $Z_L = Z_1 + Z_2$

$$= j\,30 + 60$$

Reflection coefficient

$$\rho = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\rho = \frac{60 + j\,30 - 60}{60 + j\,30 + 60}$$

$$= \frac{j\,30}{120 + j\,30} = \frac{j\,1}{4 + j\,1}$$

$$|\rho| = \frac{1}{\sqrt{16 + 1}} = \frac{1}{\sqrt{17}}$$

$$VSWR = \frac{1 + |\rho|}{1 - |\rho|} = \frac{1 + \frac{1}{\sqrt{17}}}{1 - \frac{1}{\sqrt{17}}}$$

= 1.64

Option (b)

21. A transmission line of characteristic impedance 50 Ω is terminated in a load impedance Z_L. The VSWR of the line is 5 and the first of the voltage maximum in the line is observed at a distance of $\frac{\lambda}{4}$ from the load. The value of Z_L is

(a) $10\ \Omega$ (c) $250\ \Omega$

(b) $(19.23 + j46.15)\Omega$ (d) $(19.23 - j46.15)\Omega$

[GATE 2011: 2 Marks]

Soln. For a transmission line, $Z_0 = 50\ \Omega\ , VSWR = 5$

Distance of the first voltage maximum from the load $= \frac{\lambda}{4}$. The distance between adjacent maxima and minima should be $\frac{\lambda}{4}$ in a standing wave pattern, V_{min} should occur at the load.

V_{min} occurs for a resistive termination. V_{min} occurs at load if

$$Z_L = Z_{min} = \frac{Z_0}{S} \qquad Z_L = \frac{Z_0}{S} = \frac{50}{5} = 10\Omega$$

Option (a)

22. A transmission line with a characteristic impedance of 100 Ω is used to match a 50 Ω section to a 200 Ω section. If the matching is to be done both at 429 MHz and 1 GHz. The length of the transmission line can be approximately.

(a) 82.5 cm (c) 1.58 m

(b) 1.05 m (d) 1.75 m

[GATE 2012: 2 Marks]

Soln. $$Z_0 = \sqrt{Z_1 Z_2}$$

$$100 = \sqrt{50 \times 200}$$

50 Ω 100 Ω 200 Ω

Z_1 ——— Z_0 ——— Z_2

This is quarter wave matching. The length would be odd multiples of $\lambda/4$.

$$l = (2m + 1)\lambda/4$$

$$f_1 = 429\ MHz$$

$$\frac{\lambda_1}{4} = \frac{3 \times 10^8}{429 \times 10^6 \times 4}$$

$$\ell_1 = 0.174\ m$$

$$f_2 = 1\ GHz$$

$$\frac{\lambda_2}{4} = \frac{3 \times 10^8}{10^9 \times 4} = 0.075\ m$$

$$\ell_2 = 0.075\ m$$

$$(2m+1) = \frac{1.58}{\ell_1} = \frac{1.58}{0.174} = 9$$

$$(2m+1) = \frac{1.58}{\ell_2} = \frac{1.58}{0,075} = 21$$

Only option (c) is odd multiples of both ℓ_1 and ℓ_2 .

23. The input impedance of a $\lambda/8$ section of a lossless transmission line of characteristic impedance 50 Ω is found to be real when the other end is terminated by a load $Z_L = R + jX$. If X is 30 Ω, the value of R is _____.

[GATE 2014: 2 Marks]

Soln.

$$Z_{in} = Z_0 \left(\frac{Z_L + j\,Z_0 \tan\beta l}{Z_L + j\,Z_L \tan\beta l} \right)$$

$$l = \lambda/8$$

$$\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{8}$$

$$= \frac{\pi}{4}$$

$$\tan(\beta l) = \tan\left(\frac{\pi}{4}\right) = 1$$

$$Z_{in} = Z_0 \left[\frac{Z_L + j\,Z_0}{Z_0 + j\,Z_L}\right]$$

$$Z_L = R + jX, \qquad Z_0 = 50\Omega$$

$$= R + j\,30$$

$$Z_{in} = 50\left[\frac{R + j30 + j50}{50 + j\,(R + j\,30)}\right]$$

$$= 50\left[\frac{R + j80}{(50 - 30) + jR}\right]$$

$$Z_{in} = 50\left[\frac{R + j80}{20 + jR}\right]$$

$$= 50\left[\frac{(R + j80)(20 - jR)}{20^2 + R^2}\right]$$

Since only real part of Z_{in} exists so imaginary part of $Z_{in} = 0$

$$Z_{in} = 50\left[\frac{20R + 1600j - j\,R^2 + 80\,R}{20^2 + R^2}\right]$$

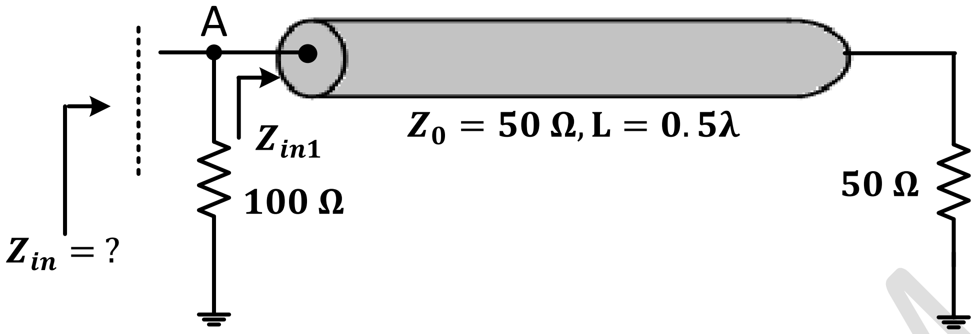
$$Z_{imaginary} = \frac{1600 - R^2}{20^2 + R^2}$$

$$1600 - R^2 = 0$$

$$R^2 = 1600$$

$$R = 40\,\Omega$$

24. In the transmission line shown the impedance Z_{in} between node A and the ground is



Soln.

Since line is of length 0.5 λ

$$Z_{in_1} = Z_0 = \left[\frac{Z_L + j\,Z_0 \tan \beta l}{Z_0 + j\,Z_L \tan \beta l}\right]$$

$$for\ \lambda/2\ line\ \beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{2}$$

$$= \pi$$

$$Z_{in_1} = Z_0 \left[\frac{Z_L + j_0}{Z_0 + j_0}\right]$$

$$Z_{in_1} = Z_L = 50\Omega$$

$$Z_{in} = 100 \| 50$$

$$= \frac{100 \times 50}{150}$$

$$= 33.3\Omega$$